The Nexus of Rollover and Interest Rate Risks in Sovereign Default Models*

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Abstract

This paper proposes a model of sovereign default that features interest rate multiplicity driven by rollover risk. Our core mechanism shows that the possibility of a rollover crisis by itself can lead to high interest rates, which in turn reinforces the rollover risk. By exploiting a complementarity between the traditional notions of slow- and fast-moving crises, our model generates rich simulated debt dynamics that feature frequent defaults and a volatile bond spread even in the absence of any shocks to fundamentals. With risky income, our mechanism amplifies the dynamics of debt and spreads relative to the literature benchmarks.

Keywords: Sovereign default, self-fulfilling crises, multiple equilibria

JEL Classification Numbers: E44, F34

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1 Introduction

Sovereign debt crises are associated with significant costs and disruption to economic activity. Such events are often preceded by rising interest rates and debt issuances, which makes it increasingly difficult for countries to finance the higher debt service. At the heart of the debate lies the question of whether debt crises are driven by changes in fundamentals (i.e., recessions) or by coordination failures among creditors. In this paper, we develop a quantitative sovereign debt model with self-fulfilling debt crises, in which interest rate spreads and debt issuance exhibit realistic dynamics even in the absence of fluctuations in fundamentals. We explore the complementarity between the rollover risk in Cole and Kehoe (2000) and the interest rate risk in Calvo (1988). The higher probability of a rollover crisis in the future leads to higher interest rates today, which in turn increases the likelihood of a rollover crisis in the future and justifies the high interest rates.

The rollover risk in Cole and Kehoe (2000) is motivated by the fact that countries often rely on new borrowing to finance current debt service. If creditors expect an imminent default on the previously issued debt, the price of newly issued bonds is low, which indeed pushes the country to default on the maturing debt because it cannot raise enough revenue. Models of rollover risk have been used to explain fast reversal in capital inflows at the height of a debt crisis, such as December 1994 in Mexico or the summer of 2012 in Europe.[1] A crucial assumption in these models concerns the timing of actions, with the borrower issuing new debt before deciding whether to default on previously issued debt.

In contrast, the interest rate risk in Calvo (1988) is motivated by the fact that higher interest rates can by themselves lead to a buildup in debt service and leave countries at risk of default. Models with this kind of interest rate multiplicity have gained renewed interest recently in an effort to explain the episodes in which bond spreads increase slowly over time, as in Europe starting from 2008.[2] An important assumption in these models concerns the type of actions the borrower can take. We assume the borrower decides on the amount of revenue it needs to raise from bond markets. Therefore, the amount to be repaid in the future is market determined and subject to self-fulfilling crises.[3] High interest rates make default more likely, which in turn justifies the high interest rates.

[1]This source of multiplicity has been explored in Cole and Kehoe (1996), Conesa and Kehoe (2017), Bocola and Dovis (2019), Aguiar et al. (2022), and Bianchi and Mondragon (2022).

[2]This source of multiplicity has been explored in Aguiar and Amador (2020), Lorenzoni and Werning (2019), and Ayres et al. (2018, 2023).

[3]An alternative is to assume the borrower chooses the amount to be repaid in the future, as in Arellano (2008), which leads to equilibrium uniqueness. See Ayres et al. (2018) and Lorenzoni and Werning (2019).

The European debt crisis, however, was marked by both a gradual buildup in interest rate spreads and eventually a fast reversal in capital inflows. Because it is challenging to build a unified theory of the events of 2008-2012 using one of the traditional approaches to self-fulfilling debt crises alone, without the need of further assumptions on income processes or preferences, our paper proposes to combine them. We exploit a complementarity between the rollover and interest rate risks by adopting both the timing assumption in Cole and Kehoe (2000) and the assumption on borrower actions in Calvo (1988). As a result, we obtain a simple framework where the self-fulfilling buildup of interest rates pushes the government endogenously into a rollover crisis zone and leaves it at the mercy of further market sentiments. In such cases, interventions by a lender of last resort are capable of ruling out the high spreads, making our framework consistent not only with the weak correlation between spreads and fundamentals throughout the crisis, but also with the interpretation that policy announcements by the European Central Bank in the summer of 2012 were enough to lower spreads without any intervention in bond markets taking place in practice.[4]

We begin by illustrating our core mechanism in a simple three-period model where the borrower issues Calvo-type debt in the first period and faces a Cole-Kehoe-style rollover risk in the second period. Without any income fluctuations and for a risk-neutral borrower, we show analytically that this setup can produce interest rate multiplicity that is generated by the rollover risk. If creditors expect a rollover crisis in the second period, they will charge a higher interest rate on the debt issued in the first period. But this larger debt payment in the second period may in turn push the borrower to default in case a rollover crisis occurs, hence justifying the higher initial interest rate.

Guided by the results from our stylized three-period model, we proceed to test the complementarity between the two notions of multiplicity in a more typical infinite-horizon setup with one-period debt.[5] We maintain the assumption of no income shocks; hence, the only source of risk comes from the two types of “sunspot" variables that exogenously drive the rollover and interest rate risk sentiments. We find that the model can generate interesting dynamics of the simulated debt and bond spread, which stems from an interaction between the two types of multiplicity. Specifically, every simulated path starts

[4]These were the main features used to support the view of an expectations-driven crisis in Europe, similarly to the debt crisis in Mexico in 1994.

[5]As elaborated in the literature review section, while we limit the scope of this paper to the most natural case of short-term debt, the main results extend to the version of our model with a moderate profile of long-term debt.

with a “slow-moving debt crisis," where the borrower’s debt accumulation is propelled by the creditors’ coordination on the high interest rate (a bad Calvo sunspot). Then, faced with high debt and a high interest rate burden, the borrower finds himself in a crisis zone and remains vulnerable to a rollover crisis (a bad Cole-Kehoe sunspot), which ultimately causes him to default. Alternatively, a temporary and unexpected intervention by a lender of last resort has the power to bring the spread down to zero, but this does not lead to a debt reduction and an exit from the crisis zone. We show that the model simulations produce a realistic debt ratio and a high and volatile bond spread. A comparative statics exercise reveals that we can obtain a host of different combinations of these salient moments by varying the probability of a bad Calvo sunspot.

Finally, we augment the model with income shocks and show that the feedback loop between both types of multiplicity amplifies the dynamics of debt crises relative to the benchmark models that admit each one of them separately. In particular, the model with both sources of multiplicity attains the high average spread, as in the pure Cole-Kehoe setup, and simultaneously generates a high standard deviation of the spread. By contrast, the pure Cole-Kehoe variant of the model features a high average spread with zero volatility, whereas the pure Calvo variant can produce positive combinations of the two moments, but their magnitude is less than half as high as in the baseline. The presence of income shocks also widens the interval of probabilities of a rollover crisis for which the two sunspots interact, relative to the model that features non-fundamental shocks only. Income shocks also allow for interesting dynamics of debt and spread that goes in both directions (increases and reductions). As such, the addition of income shocks increases the relevance of our core mechanism.

1.1 Literature review

This paper is closely related to the sovereign default literature with self-fulfilling debt crises. The main two papers that lay foundations for our approach are Calvo (1988) and Cole and Kehoe (2000). Our contribution is to show that a complementarity exists between them. In a closely related paper, Corsetti and Maeng (2020) also study a model with both types of multiplicity to show that the two types of debt crises can arise in a unified framework for different state variables. By contrast, our paper highlights a complementarity between the two modeling assumptions and studies the quantitative implications of their interaction, in particular in a framework with no additional sources of uncertainty.

Our results are also related to the recent papers that assume equilibrium multiplicity in the spirit of Calvo – Lorenzoni and Werning (2019) and Ayres et al. (2018, 2023), among others – as well as those based on the Cole-Kehoe timing – Conesa and Kehoe (2017), Bocola and Dovis (2019), Stangebye (2020), Aguiar et al. (2022), and Bianchi and Mondragon (2022), among others. Several of these papers also focus on equilibrium multiplicity in the presence of long-term debt. Long-term bonds naturally limit the applicability of rollover crises and open the door to other forms of multiplicity, as shown in Aguiar and Amador (2020). For this reason, we focus on the case of one-period debt.[6]

Our paper contributes to our understanding of the forces at play during the European debt crisis. Using models based on fundamentals, Paluszynski (2023) explains the gradual development of that episode, while Paluszynski and Stefanidis (2023) show that frictions in spending adjustment may explain why governments simultaneously increased their external debt. The present paper achieves similar objectives in a model where the debt crisis is self-fulfilling.

2 Multiplicity in a three-period model

This section presents a simple three-period environment to illustrate our core mechanism. For simplicity, we present the derivation of our main result for a risk-neutral borrower. In the quantitative section, we assume a risk-averse borrower.

The borrower receives *deterministic* endowment y in all three periods ($t = 0, 1, 2$). It has zero initial debt and can issue one-period non-contingent bonds to competitive risk-neutral lenders. The borrower is not committed to repay the debt. In the case of default, he is permanently excluded from international financial markets and restricted to consume $y^d < y$. The risk-free gross interest rate is denoted by R^*. To induce borrowing, we assume the borrower has a lower discount factor than the lenders, denoted by β.

As in Cole and Kehoe (2000), we assume the borrower chooses whether or not to default on the previously issued debt *after* the new debt issuance takes place.[7] In this setting, lenders may not roll over the debt if the lack of new borrowing pushes the borrower to default on the old debt, which characterizes the rollover risk. As in Eaton and Gersovitz

[6] While the main results extend to the version of our model with a moderate profile of long-term debt, a comprehensive analysis of that variant is beyond the scope of this paper.

[7] We do not allow for randomization over the default decision.

(1981), we assume that when the bond auction takes place, the borrower moves first by committing to the amount of resources he wishes to raise in the current period, denoted by b. Lenders move next and set the gross interest rate R. These assumptions generate the interest rate multiplicity, as in Calvo (1988).[8] For a given b, a higher R increases the probability of default because it increases the debt service. In turn, a higher probability of default implies a higher R, as lenders must have an expected return equal to R^* in equilibrium.

We present and solve the problem backward. In period $t = 2$, the only choice for the borrower is whether to repay the debt issued in the previous period, $R_2 b_2$, or to default. The borrower defaults if $y^d > y - R_2 b_2$ and repays otherwise.

It follows that in period $t = 1$, if the lenders roll over the debt, the interest rate is uniquely determined. Let us define the threshold $\widetilde{B}_2 \equiv \left(y - y^d\right)/R^*$. For $b_2 \leq \widetilde{B}_2$, there is no default and R_2 must be equal to R^*. For $b_2 > \widetilde{B}_2$, the borrower defaults for sure, so $\widetilde{B}_2$ becomes a borrowing limit.

If lenders do not roll over the debt in $t = 1$, the borrower defaults if

$$v_1^d \equiv (1+\beta) y^d > y - R_1 b_1 + \beta y,$$

where $R_1 b_1$ is the debt service on the debt issued in $t = 0$.[9] A rollover crisis may happen only if it pushes the country to default, so the borrower is subject to rollover risk only if

$$R_1 b_1 > (1+\beta)(y - y^d). \tag{1}$$

Note that a rollover crisis is equivalent to setting the borrowing limit to zero, a convention we will adopt to simplify the exposition. We can express the problem in period $t = 1$ as

$$v_1\left(R_1 b_1, s_{ck}\right) = \max\{v_1^{nd}\left(R_1 b_1, s_{ck}\right), v_1^d\},$$

where

$$v_1^{nd}\left(R_1 b_1, s_{ck}\right) = \max_{b_2 \leq \overline{B}_2(R_1 b_1, s_{ck})} y - R_1 b_1 + b_2 + \beta\left(y - R^* b_2\right).$$

The sunspot variable $s_{ck} \in \{0, 1\}$ commands the Cole-Kehoe type of market sentiment.

[8] See Ayres et al. (2023).

[9] As in Aguiar et al. (2016), we assume the borrower does not keep the proceeds from the new bond auction in case it defaults on the old debt.

If $s_{ck} = 0$ and condition (1) holds, a rollover crisis happens and the borrowing limit $\overline{B}_2(R_1 b_1, s_{ck})$ equals zero. Otherwise, $\overline{B}_2(R_1 b_1, s_{ck}) = \widetilde{B}_2$.[10] In addition, note that the condition for the rollover risk in (1) depends on R_1, which gives rise to interest rate multiplicity. For a given b_1, higher R_1 makes a rollover crisis more likely. In turn, the higher probability of a rollover crisis implies a higher R_1.

We now turn to the borrower's problem in $t = 0$:

$$v_0(s_c) = \max_{b_1 \leq \bar{b}_1} y + b_1 + \beta \sum_{s_{ck} \in \{0,1\}} \pi(s_{ck})\, v_1\Big(R_1(b_1, s_c) b_1, s_{ck}\Big),$$

where $\pi(s_{ck})$ denotes the probability distribution over the values that s_{ck} may take in $t = 1$. We let p denote the probability of the bad sunspot, $\pi(0) = p$. The state variable $s_c \in \{0, 1\}$ denotes the Calvo-type sunspot. In case there are multiple interest rates for a given b_1 such that lenders receive an expected return equal to R^*, we use the sunspot variable s_c as a device to select the interest rate. As in Ayres et al. (2023), we will focus on two extreme cases. In the bad sunspot state, $s_c = 0$, R_1 takes the highest possible value. In the good sunspot state, $s_c = 1$, R_1 takes the lowest possible value. Lemma 1 characterizes all pairs (b_1, R_1) such that lenders receive return R^* in expectation.

Lemma 1 *The pairs (b_1, R_1) in which lenders receive an expected return equal to R^* given the borrower's optimal borrowing and default strategies are:*

(i) $b_1 \leq \frac{(1+\beta)(y-y^d)}{R^*} \equiv \underline{\underline{B}}_1$ *and* $R_1 = R^*$.

(ii) $\underline{B}_1 \equiv \frac{(1+\beta)(y-y^d)(1-p)}{R^*} \leq b_1 \leq (1-p)(y-y^d)\left(\frac{1}{R^*} + \frac{1}{(R^*)^2}\right) \equiv \overline{B}_1$ *and* $R_1 = \frac{R^*}{1-p}$.

Proof Recall that the borrower will default in period $t = 1$ if $R_1 b_1 > (1+\beta)(y - y^d)$. Because there are two possible values for the interest rate, R^* and $R^*/(1-p)$, we have two debt thresholds that limit the repayment decision in the case of low and high interest rates:

$$b_1 \leq \frac{(1+\beta)(y-y^d)}{R^*} \equiv \underline{\underline{B}}_1$$
$$b_1 \leq \frac{(1+\beta)(y-y^d)(1-p)}{R^*} \equiv \underline{B}_1.$$

Finally, we need to find a debt threshold that makes the borrower indifferent between

[10] Note that the optimal strategy for the borrower in this simple case is to set $b_2 = \overline{B}_2(R_1 b_1, s_{ck})$.

repaying and defaulting when markets are open in period $t = 1$. The condition is

$$v_1^d = (1+\beta)y^d = y - \frac{R^*}{1-p} b_1 + b_2 + \beta \max\{y - R_2 b_2, y^d\}.$$

Under risk neutrality, assuming the borrower is impatient enough, the optimal borrowing in that period is $b_2^* = \frac{y-y^d}{R^*}$. Plugging this value into the indifference condition yields the following upper debt threshold:

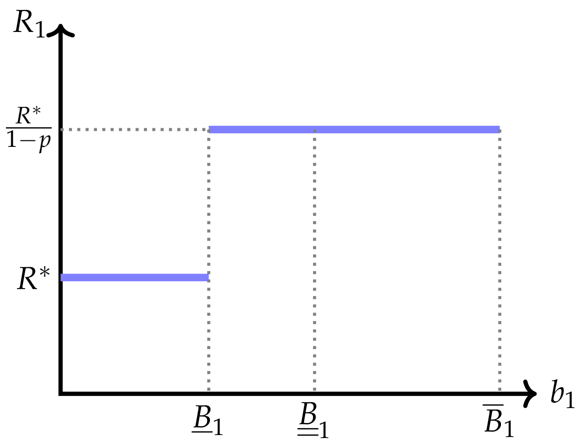
$$b_1 = (1-p)(y-y^d)\left(\frac{1+R^*}{R^{*2}}\right) \equiv \overline{B}_1.$$

■

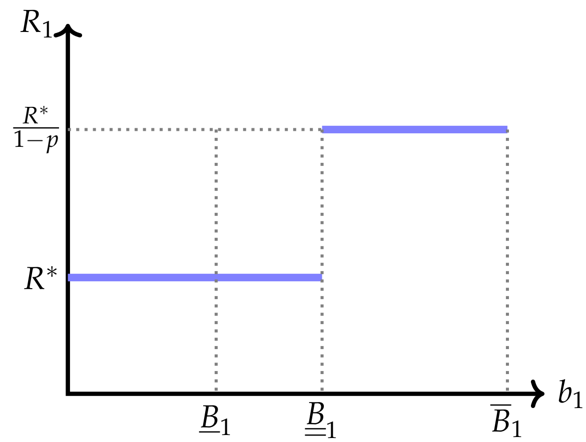
For any debt level smaller than $\underline{B}_1$, the borrower repays even if lenders do not roll over the debt. Hence, the interest rate is unique and equal to the risk-free rate. For a level of borrowing between $\underline{B}_1$ and $\underline{\underline{B}}_1$, however, multiple interest rates arise. It is noteworthy that, in this case, it is the rollover risk in period $t = 1$, rather than the income shock, that drives the multiplicity of the interest rate. In other words, for a given $b_1 \in \left[\underline{B}_1, \underline{\underline{B}}_1\right]$, a high or low interest rate R_1 determines whether or not the borrower finds itself in a Cole-Kehoe type of crisis zone. Finally, if the debt level is sufficiently high, above $\underline{\underline{B}}_1$, then the interest rate is again unique and equal to the high rate $\frac{R^*}{1-p}$. At such debt levels, the borrower is in the crisis zone unconditionally and defaults in the event of a bad s_{ck} sunspot even if interest rates were set to R^*.

The borrower observes s_c before the bond auction and internalizes how the interest rate will vary with respect to the amount of debt it chooses to issue. Therefore, when choosing how much to borrow, he considers an interest rate schedule $R_1(b_1, s_c)$ as a mapping from debt levels into unique interest rate values. Figure 1 presents a stylized illustration of the two interest rate schedules the borrower may face. Panel 1(a) features the case of the higher rate within the multiplicity interval, $R_1(b_1, 0)$, while Panel 1(b) features the case of the lower rate, $R_1(b_1, 1)$.

In Appendix A, we extend Lemma 1 to the case of a risk-averse borrower. Risk aversion further enhances the interest rate multiplicity, but it is no longer possible to find closed-form solutions for all debt thresholds. In addition, while the present three-period model mainly serves to illustrate our mechanism, in Appendix A.2 we also present a numerical example where we assume reasonable parameter values and plot the resulting interest



(a) High interest rate schedule $R_1(b_1, 0)$



(b) Low interest rate schedule $R_1(b_1, 1)$

Figure 1: Stylized illustration of the interest rate schedules

rate schedules and policy functions. In Section 3, we extend the model to an infinite horizon and show that the rollover multiplicity is quantitatively significant.

3 Infinite-horizon model

In this section, we develop an infinite-horizon model to study the interaction between interest rate multiplicity and rollover risk.

3.1 Economic environment

Consider a small open economy with a benevolent sovereign that borrows internationally from competitive lenders and receives a stochastic endowment. Time is discrete and indexed by $t = 0, 1, 2, ...$ Markets are incomplete, and the only asset available for trading is the one-period non-contingent bond. The risk-free gross interest rate is R^*. The representative household has preferences given by the expected utility of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2}$$

where we assume the function $u(\cdot)$ is strictly increasing, concave, and twice continuously differentiable. The discount factor is given by $\beta \in (0, 1)$.

Income process The economy's income is affected by stochastic endowment growth realizations and evolves according to

$$Y_t = g_t Y_{t-1}, \tag{3}$$

where g_t denotes the growth shock. The growth rate can take two values, g_L and g_H, with $g_H > g_L$. It follows a Markov process with the transition probability matrix given by

$$\Pi = \begin{bmatrix} \pi_L & 1-\pi_L \\ 1-\pi_H & \pi_H \end{bmatrix}, \tag{4}$$

where $Pr(g_{t+1} = g_L | g_t = g_L) = \pi_L$ and $Pr(g_{t+1} = g_H | g_t = g_H) = \pi_H$. We model the income shock process as growth regimes following Ayres et al. (2018), who show that a bimodal income process generates Calvo-style interest rate multiplicity.[11] That said, our main quantitative result in Section 3.3 is obtained with a variant of the model with no income shocks whatsoever.

Timing The timing assumptions are the same as in Section 2. The borrower chooses whether or not to default on the debt from the previous period *after* the new debt issuance (Cole and Kehoe, 2000). Similar to Calvo (1988), when the bond auction takes place, the borrower moves first by committing to the amount of revenue it wishes to raise from bond markets in the current period, b. Lenders move next and set the gross interest rate R. Shocks are observed at the beginning of the period.

States The state variables are $\{A, Y, \mathbf{s}\}$. $A = RB$ denotes the total debt service to be paid in the current period, Y is the current income, and $\mathbf{s} = \{s_c, s_{ck}\}$ is a vector of sunspot realizations corresponding to the interest rate multiplicity and rollover risk, respectively.

Recursive problem The value function of the government involves a choice of whether or not to default:

$$V(A, Y, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ (1-d)V^{nd}(a, Y, \mathbf{s}) + dV^d(Y, \mathbf{s}) \right\}.$$

[11] It is also possible for the model to feature a transitory shock, but its variance cannot be too large.

The value associated with repayment is

$$V^{nd}(A,Y,\mathbf{s}) = \max_{B' \leq \overline{B}(A,Y,\mathbf{s})} \left\{ u(C) + \beta \sum_{y'} \sum_{\mathbf{s}'} \Pi(y'|y) p(\mathbf{s}'|\mathbf{s}) V\Big(B'R(B',Y,\mathbf{s}), y', \mathbf{s}'\Big) \right\}$$

subject to

$$C = Y - A + B'.$$

The value associated with default is

$$V^d(Y,\mathbf{s}) = u\Big(Y(1-\phi)\Big) + \beta \sum_{y'} \sum_{\mathbf{s}'} \Pi(y'|y) p(\mathbf{s}'|\mathbf{s}) \Big\{ \theta V\Big(0, y', \mathbf{s}'\Big) + (1-\theta) V^d\Big(y', \mathbf{s}'\Big) \Big\},$$

where ϕ represents the fraction of income lost upon default. We assume that, following a default, the borrower has probability θ of being readmitted to capital markets in each period and the recovery rate of defaulted debt is zero.

As in Section 2, the borrowing limit $\overline{B}\,(A,Y,\mathbf{s})$ equals zero whenever $s_{ck} = 0$, and the lack of new borrowing pushes the country to default. That happens when the following condition is satisfied:

$$u(Y-A) + \beta \sum_{y'} \sum_{\mathbf{s}'} \Pi(y'|y) p(\mathbf{s}'|\mathbf{s}) V\Big(0, y', \mathbf{s}'\Big) \leq V^d(Y,\mathbf{s}).$$

Definition 2 formally defines an equilibrium in this economy.

Definition 2 *A Markov perfect equilibrium for this economy consists of the government value functions $V(A,Y,\mathbf{s})$, $V^{nd}(A,Y,\mathbf{s})$, $V^d(Y,\mathbf{s})$; policy functions $B'(A,Y,\mathbf{s})$ and $d(A,Y,\mathbf{s})$; the interest rate schedule $R(B',Y,\mathbf{s})$ and the borrowing limit function $\overline{B}\,(A,Y,\mathbf{s})$ such that:*

1. *Policy function $d(A,Y,\mathbf{s})$ solves the government's default-repayment problem.*

2. *Policy functions $B'(A,Y,\mathbf{s})$ solve the government's consumption-saving problem.*

3. *Interest rate schedules $R(B',Y,\mathbf{s})$ and borrowing limit functions $\overline{B}\,(A,Y,\mathbf{s})$ are such that international lenders receive an expected return equal to R^*.*

3.2 Quantification of the model

In this section, we parameterize our model in order to evaluate its performance quantitatively. We do not conduct an independent calibration of the structural parameters, but instead we adopt them directly from Aguiar et al. (2022) and use them for all variants of our model presented in the following sections. The parameter values are as follows. The risk-free rate r is set to 0.01, the risk aversion γ is 2, the discount factor β is 0.8, the income loss in default ϕ is 0.03, and the probability of re-entry following a default θ is 0.125. These parameter values replicate the typical behavior of the Mexican government in terms of its borrowing and defaulting decisions.

To estimate the Markov-switching process for growth shocks, we use Mexico's GDP data for the period 1980-2021. We use the filter in Kim (1994) and estimate the parameters by maximum likelihood estimation. The resulting parameter estimates are as follows. The high and low regime growth rates are $g_h = 1.02$ and $g_l = 0.96$, while the persistence of high and low regimes is $\pi_H = 0.8$ and $\pi_L = 0.3$, respectively.

3.3 No growth shocks

As a first step, we evaluate the model with no growth regimes (and thus, no fundamental shocks whatsoever). Income is deterministic and equal to 1 in every period. Hence, in this variant of the model, rollover risk is the sole driver of defaults and a potential interest rate multiplicity. We start with 0.1 as the initial probability of the bad Calvo sunspot realization ($s_c = 0$), and we vary the probability of the Cole-Kehoe sunspot to illustrate how the model works. Table 1 presents the statistics from a simulated ergodic distribution for three different probabilities of a bad Cole-Kehoe sunspot ($s_{ck} = 0$). As is evident, the model features starkly different types of behavior for seemingly similar values of this parameter. When the probability is about 5.6% or lower, the agent borrows on the higher interest rate schedule and defaults every time a rollover crisis occurs. As a result, the average spread is roughly equal to the probability of a bad Cole-Kehoe sunspot, while the variance of the spread is zero. On the other hand, for a probability of about 5.8% or higher, the agent borrows on the lower interest rate schedule and reduces debt every time the Calvo sunspot switches to bad in order to avoid the region of multiplicity. As a result, no defaults occur on the equilibrium path and the bond spread is exactly zero. In between the two extremes, there is an interval of the bad Cole-Kehoe sunspot probabilities around the value of 5.7% where an interesting action occurs. In this case, the agent initially borrows on the lower interest rate schedule but then increases the debt and jumps to the higher rate when the Calvo sunspot switches to bad (a "slow-moving debt crisis"). The borrower remains there until a Cole-Kehoe-type rollover crisis forces him into default. It should be emphasized that while the interval of sunspot probabilities for which the interesting behavior occurs is quite narrow, it is so because the model does not feature any other sources of uncertainty. Section 3.4 shows that this interval widens considerably when realistic income shocks are introduced. It is also noteworthy that the average debt-to-income ratio in this case, an untargeted moment, comes out exactly equal to its empirical counterpart of 66%, as reported by Aguiar et al. (2022) (for quarterly data).

Table 1: Simulated results with no growth regimes

$\mathbf{P(s_{ck} = 0)}$	0.056	0.057	0.058
E(d/Y)	18.2	16.5	13.1
E(s)	6.0	4.4	0.0
σ(s)	0.0	2.7	0.0
ρ(s,TB)	0.0	-0.59	0.0

Note: d=debt, Y=income, s=spread, TB=trade balance (relative to income), $s_{ck} = 0$ denotes a bad Cole-Kehoe-type sunspot realization.

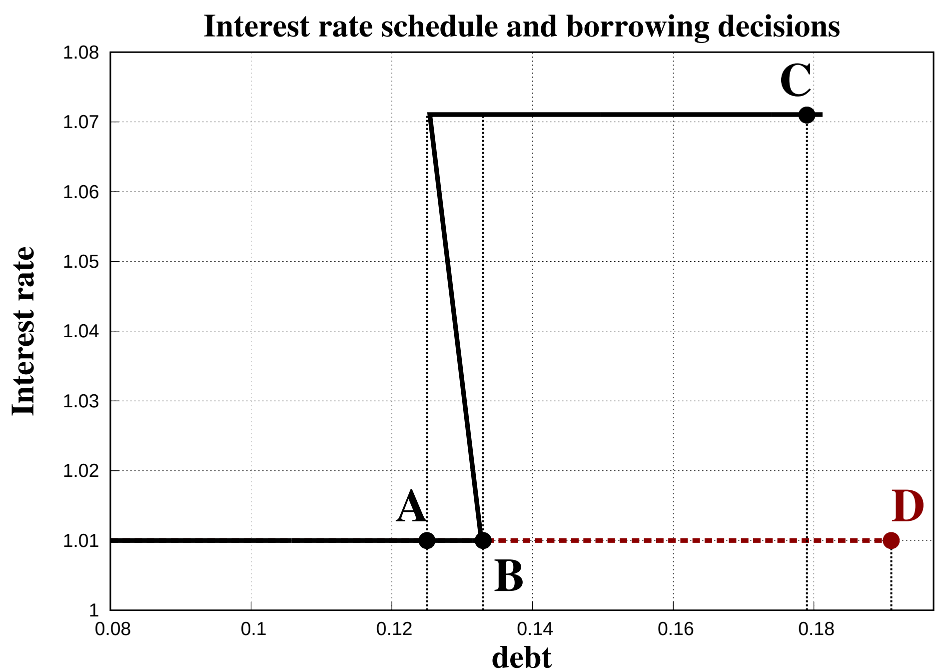
Figure 2(a) plots the interest rate schedule in the intermediate case of $P(s_{ck} = 0) = 0.057$. The clear multiplicity interval confirms our analytical result from Section 2, which shows that the Calvo action space can combine with rollover crises to generate overlapping interest rate schedules *with no income shocks*. The graph also describes the dynamics of the borrower's decisions in this model. As the agent accumulates debt starting from zero, he moves along the risk-free interest rate toward the points labeled "A" and "B." The former is chosen if the Calvo sunspot realization is initially bad, whereas the latter is eventually selected when the realization switches to good. Once the borrower lands at point B, he will not retreat to point A upon another bad Calvo sunspot, but instead will borrow all the way to point C and incur an interest rate spread of 6%. With no additional friction or shocks in the model, the agent stays at point C until a Cole-Kehoe rollover crisis occurs, in which case he defaults.[12] Appendix B provides further analysis of this behavior by examining the entire policy functions corresponding to each column of Table 1.

Figure 2(a) also features point D which we refer to as a policy intervention. Suppose that a lender of last resort unexpectedly steps in and eliminates the possibility of a rollover crisis for one period.[13] In such case, the interest rate spread drops to zero in that given period while the borrower increases the face value of the debt slightly to keep the debt obligation

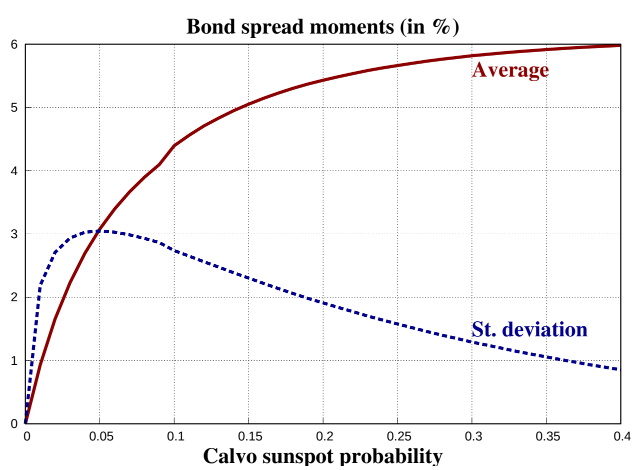
[12] As Section 3.4 shows, the full version of this model with stochastic growth regimes also features endogenous debt reductions.

[13] In the case of a permanent elimination of a rollover crisis, the model becomes degenerate as the impatient government borrows up to the unique debt limit and never defaults.

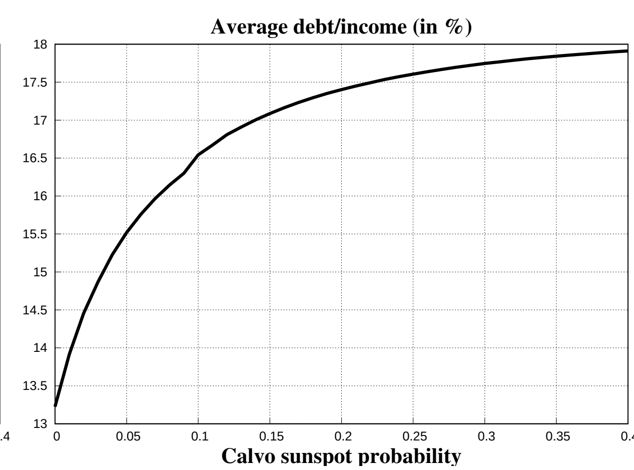
constant. The effects of such an intervention, reminiscent of the events at the height of the European debt crisis in the summer of 2012, are specific to models with the Cole-Kehoe-type rollover crises. If the intervention were instead focused on temporarily eliminating the possibility of a bad Calvo sunspot, it could prevent the initial spread buildup, but it would have no ability to *bring it down*. This shows that our model can simultaneously generate features of the initial (slow-moving) and later (fast-moving) stages of the European debt crisis, and highlights the distinct policy implications at these stages.



(a) Interest rate schedules and policy functions with no income shocks



(b) Bond spread moments



(c) Average debt ratio

Figure 2: Simulated moments as function of the Calvo sunspot probability

An interesting aspect of our model is that the interval of Cole-Kehoe sunspot probabilities that generates these dynamics is the same for any Calvo sunspot probability parameter that we choose. However, the implications for the resulting simulated moments are quite different as we vary the likelihood of a Calvo-style crisis. Panels 2(b) and 2(c) of Figure 2 explore these comparative statics by plotting the basic moments of the bond spread and debt for a range of values that this parameter can take. We find that the average spread and average debt ratio are both monotonically increasing in the probability of the bad Calvo sunspot. The intuition is simple: as switching to the higher interest rate schedule becomes more likely, the borrower spends less time at point B of Figure 2(a), characterized by lower debt and a spread of zero, and more time at point C, with high debt and a positive spread. On the other hand, the measured volatility of the spread is *non-monotonic*, initially rising sharply from zero and then falling back gradually. The intuition is straightforward: if a Calvo-style crisis is very unlikely or if it happens too often, the borrower will end up spending a disproportionate amount of time on the lower or upper interest rate schedule, respectively. Hence, there exists an intermediate value for the bad Calvo sunspot probability that balances the average time spent on the two parts of the schedule and maximizes the overall bond spread volatility. For the present parameterization, we find that the standard deviation of the bond spread peaks at around 3% for the bad Calvo sunspot probability of around 5%.

3.4 Quantitative results with growth regimes

We now evaluate our mechanism in a model with income shocks. As specified in Section 3.2, the parameters of the stochastic growth rate are based on estimating a Markov-switching process for Mexico. Table 2 presents the simulated results across four variants of our model. To offer a meaningful comparison across the different variants of the model, we adjust the sunspot probabilities so that the model exhibits a similar average debt ratio. For completeness, we also report the results of a model without interest rate multiplicity or rollover risk. In that case, there is no free parameter and debt becomes a non-targeted object averaging around twice the level in the baseline, while spreads are essentially zero for reasons similar to what Aguiar and Gopinath (2006) describe.

For our baseline model that combines interest rate multiplicity with rollover risk, we fix the Calvo sunspot probability at 10% and we use a Cole-Kehoe sunspot probability of 4.3%, which yields an average debt level of close to 16%. For the pure Calvo and pure Cole-Kehoe variants of the model, we adjust their respective probabilities of a bad sunspot realization upward so that the average debt ratios in the simulations are similar.[14]

Table 2: Simulated results in the quantitative model

Stat	Both	Only Calvo	Only C-K	None
$P(s_c = 1)$	0.1	0.275	0.0	0.0
$P(s_{ck} = 1)$	0.043	0.0	0.055	0.0
E(d/Y)	16.1%	16.1%	17.9%	30.5%
E(s)	4.0%	0.0%	5.9%	0.0%
σ(s)	1.5%	0.0%	0.0%	0.0%
ρ(TB,Y)	−0.22	−0.13	−1.0	−0.68
ρ(s,Y)	0.02	0.0	0.0	0.0
ρ(s,TB)	−0.66	0.0	0.0	0.0

Note: d=debt, Y=income, s=spread, TB=trade balance (relative to income), $s_{ck} = 0$ *and* $s_c = 0$ *denote a bad Cole-Kehoe-type and Calvo-type sunspot realization, respectively.*

As Table 2 shows, our baseline model with multiplicity and rollover risk generates a simultaneously high average and high volatility of the bond spread. Similar to the case with no fundamental shocks described in Section 3.3, a bad Cole-Kehoe sunspot is needed to trigger a run on the debt and default. However, in this model it is a high growth regime realization that propels the government to accumulate debt and enter the crisis zone under relatively high spreads. By contrast, a bad realization of the Calvo sunspot causes the opposite reaction: the debt (and spread) is reduced, and the government will repay even if markets do not open the next period. As such, our baseline model with income shocks generates interesting dynamics of debt and spreads in both directions (accumulation and reduction).

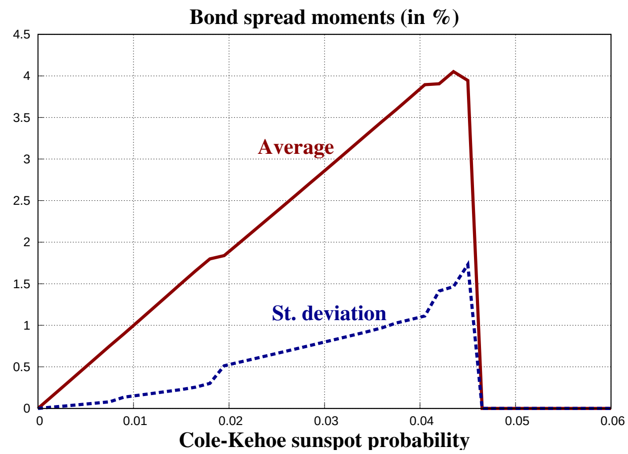
By contrast, in the pure Calvo variant of the model, a high probability of the bad sunspot is required to match the desired level of debt. At that probability, however, the borrower does not default or enter the multiplicity region on the equilibrium path resulting in a zero spread. In the pure Cole and Kehoe variant of the model, on the other hand, a level of debt close to the targeted one is attained for the probability of a bad sunspot of 5.5%, which results in roughly the same average spread (essentially, the government always enters the crisis zone and defaults if and only if the markets close). However, the consequence of this behavior is that the volatility of the spread is zero (Aguiar et al., 2022).

[14]Because of the typical knife-edge behavior of such models, it is not necessarily possible to have all three variants deliver exactly the same level of debt. Hence, we seek parameter values that bring debt levels as close to each other as possible.

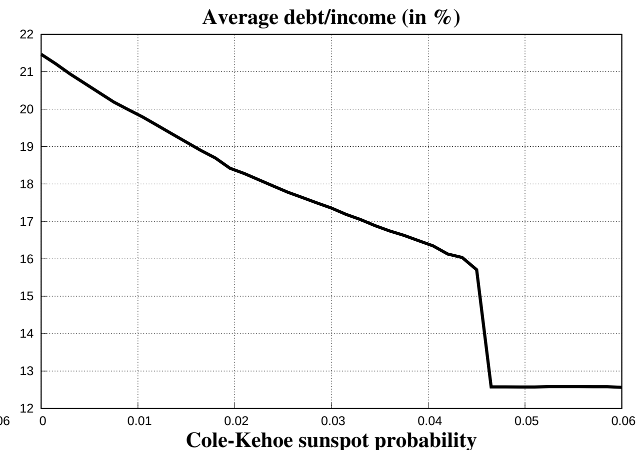
We now turn to the comparative statics analysis for our baseline model with respect to the probability of a bad Cole-Kehoe sunspot. Panels 3(a)-3(b) of Figure 3 plot the three moments of interest for the sunspot probabilities ranging up to 6%. The main thing to notice is that, in contrast to the variant of our baseline model with no income shocks (Section 3.3), the range of Cole-Kehoe sunspot probabilities for which we attain interesting debt dynamics is substantially wider. For the interval of such probabilities up to roughly 4.5%, the model generates simultaneously a high mean and high variance of the bond spread (increasing in the probability) with a realistic average debt ratio. Above that interval, the behavior of the borrower becomes less interesting; the government stays permanently outside the rollover crisis zone, and no defaults occur.

Next, we focus on the pure Calvo variant of the model to analyze its ability to generate interesting debt dynamics for a wider range of parameters. Panels 3(c)-3(d) of Figure 3 present comparative statics with respect to the probability of a bad Calvo sunspot, the only non-fundamental variable in that variant. As the plots show, defaults do occur on equilibrium path, and the model can generate a non-zero spread for sunspot probabilities lower than 7.5%, which corresponds to much higher debt-output ratios (23% and above). It is notable, however, that both the average and standard deviation of the bond spread are at least 50% smaller than in our baseline model.

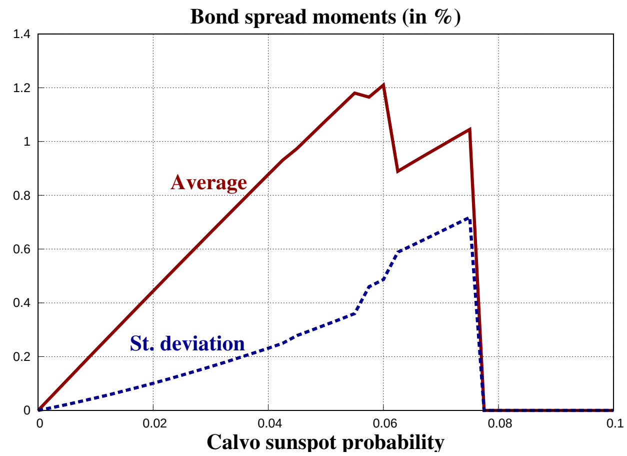
Finally, Panels 3(e)-3(f) of Figure 3 present comparative statics with respect to the bad sunspot probability in the pure Cole-Kehoe variant of the model. The government here behaves as expected: for low enough probabilities of a bad sunspot, it borrows a lot and always remains in the crisis zone, irrespective of the underlying growth regime. In all these cases, however, the mean spread corresponds directly to the assumed probability of a bad sunspot while the spread volatility is zero (unless a default has already occurred, the equilibrium spread is always a constant). For a sunspot probability greater than roughly 6%, the borrower reduces its debt sharply and stays outside of the crisis zone. As a result, both the mean and standard deviation of the spread are zero.



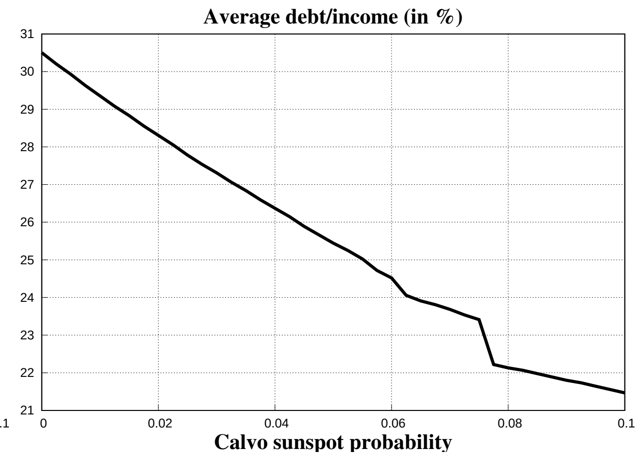
(a) Baseline model (spread)



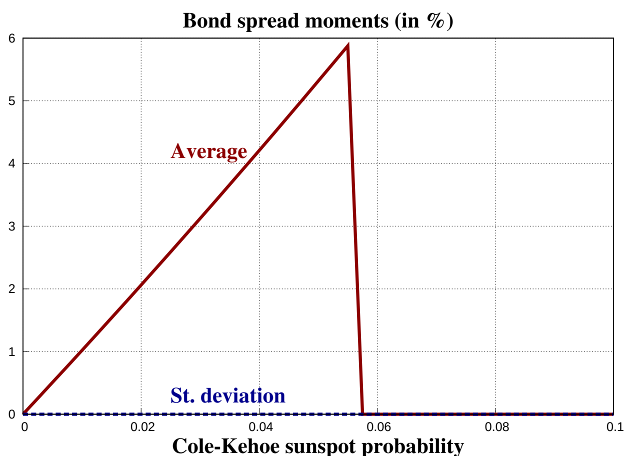
(b) Baseline model (debt)



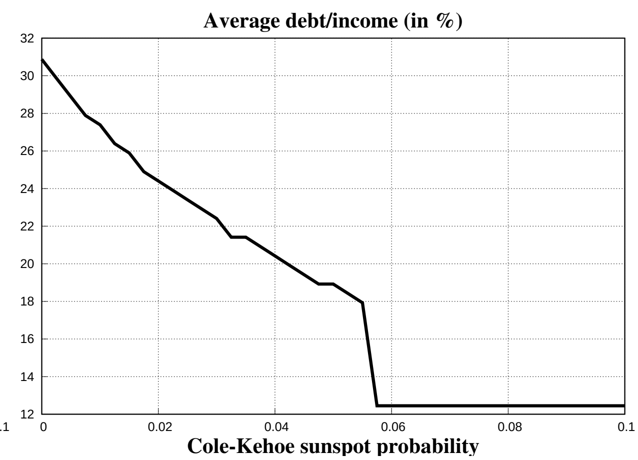
(c) Pure Calvo model (spread)



(d) Pure Calvo model (debt)



(e) Pure Cole-Kehoe model (spread)



(f) Pure Cole-Kehoe model (debt)

Figure 3: Comparative statics in the baseline model and two benchmarks: pure Calvo and pure Cole-Kehoe

4 Conclusion

This paper contributes to the literature on self-fulfilling debt crises by introducing a simple model with interest rate multiplicity generated by belief-driven runs on government debt. In turn, such runs are justified by a realization of high interest rates that by itself results from pessimistic beliefs. The main achievement of the model is to show that one can easily generate rich dynamics of sovereign debt and the interest rate spread by combining the notions of slow- and fast-moving debt crises *without any underlying shocks to fundamentals*. Through a combination of simplicity and quantitative rigor, the model allows us to simultaneously think about the slow- and fast-moving stages of the European debt crisis of 2008-2012.

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Appendices (for online publication)

A Derivations for the three-period model

In this appendix, we show that our basic three-period model result extends to the case of a risk-averse borrower. We illustrate both cases with a numerical example.

A.1 Risk aversion

We now derive the corresponding thresholds for the case of a risk-averse borrower. We assume a CRRA utility function of the form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and we analyze the problem backward. Similar to the case of risk neutrality, in period $t = 2$ the agent repays if $y - b_2 R^* \geq y^d$. In period $t = 1$, if markets do not roll over the debt, the borrower will default if

$$v_1^d = (1+\beta)u(y^d) > v_1(R_1 b_1, s_1 = 1) = u(y - R_1 b_1) + \beta u(y)$$

If $\gamma > 1$, this condition boils down to

$$R_1 b_1 > y - \left((1+\beta)(y^d)^{1-\gamma}\right)^{\frac{1}{1-\gamma}},$$

which shows that the default decision depends on the level of interest rate. Consequently, we have the two debt thresholds that limit the repayment decision for the case of low and high interest rates:

$$b_1 \leq \frac{1}{R^*}\left[y - \left((1+\beta)(y^d)^{1-\gamma} - \beta y^{1-\gamma}\right)^{\frac{1}{1-\gamma}}\right] \equiv \underline{\underline{B}}_1$$
$$b_1 \leq \frac{1-p}{R^*}\left[y - \left((1+\beta)(y^d)^{1-\gamma} - \beta y^{1-\gamma}\right)^{\frac{1}{1-\gamma}}\right] \equiv \underline{B}_1.$$

Next, to find the debt threshold that makes the borrower indifferent between repaying and defaulting when markets are open in $t = 1$, we need to find optimal borrowing b_2. Under risk aversion, this entails solving the problem

$$v_1(R_1 b_1, s_1 = 0) = \max_{b_2} \; u(y - R_1 b_1 + b_2) + \beta u(y - R^* b_2).$$

The interior solution to this problem is $b_2^* = \frac{(\beta R^*)^{-1/\gamma} y - (y - R_1 b_1)}{1 + (\beta R^*)^{-1/\gamma} R^*}$, while a corner implies $b_2^* = \frac{y - y^d}{R^*}$. To find threshold $\overline{B}_1$, we need to plug this into the indifference condition in period $t = 1$,

$$v_1^d = (1+\beta)u(y^d) = u(y - \frac{R^*}{1-p}b_1 + b_2^*) + \beta u(y - R^* b_2^*),$$

and solve for b_1. Under risk aversion, this solution cannot be obtained analytically. The results in Section A.2 present our numerical solution to this problem.

A.2 Numerical example

In this subsection, we provide a simple numerical example to show that the interest rate multiplicity characterized so far is realistic. We also extend the analysis to the case of a risk-averse borrower and show that the result becomes even stronger. The exact derivations for this case are presented in Appendix A.1.

Consider the case of a risk-averse borrower with a CRRA utility function of the form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We assume the following, fairly realistic parameterization: $\beta = 0.7$, $\gamma = 3$, $y = 1$, $y^d = 0.95$, $p = 0.8$, $R = 1.03$. Figure 4 presents the interest rate schedules, as well as the optimal debt policy for the risk-averse borrower. The solid blue line depicts the lower (risk-free) interest rate, while the dashed red and dotted blue lines represent the upper

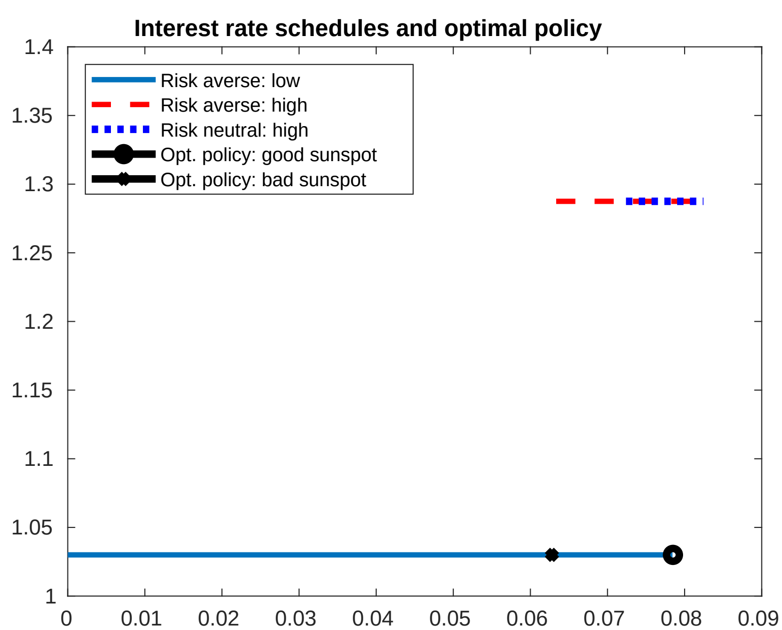


Figure 4: Interest rate schedules and optimal policy

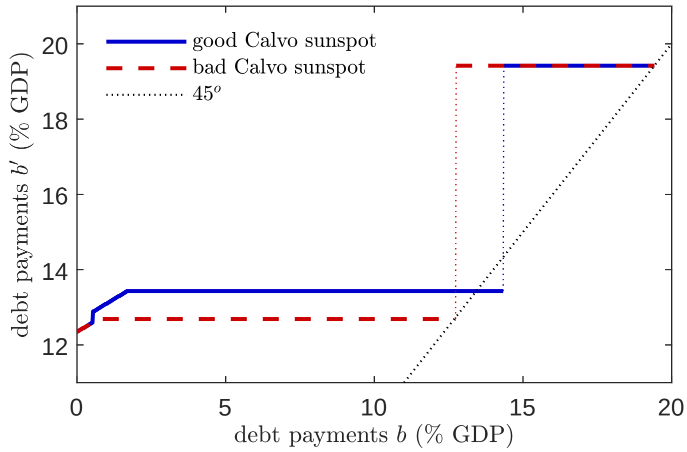
(risky) interest rate for the case of a risk-averse and risk-neutral borrower, respectively. It is immediate to notice that including risk aversion causes the interest rate multiplicity to almost double in size. The presence of this multiplicity also has real consequences for the borrower's actions. When the Calvo sunspot is bad, the government must reduce its debt by around 20%, compared to the case of a good sunspot, to avoid the higher interest rate.

B Further illustrations for the infinite-horizon model

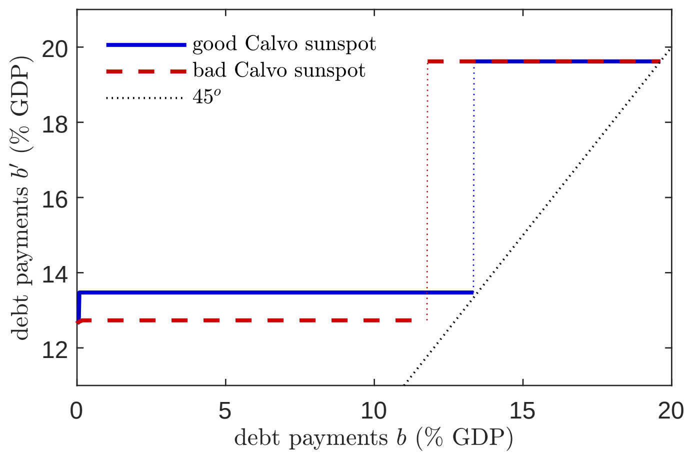
In this section, we provide further illustration of the government's borrowing choices in the model with no income shocks, corresponding to the three types of behavior presented in Table 1. Panel 5(a) of Figure 5 shows the optimal next period debt choice (b') as a function of today's debt (b), for the two possible realizations of the Calvo sunspot. As the government accumulates debt from zero, it will ultimately stop at the 45 degree line, with the exact amount depending on the current realization of the Calvo sunspot. If the sunspot is bad, then the government will stop just outside of the interval of interest rate multiplicity (corresponding to point A in Figure 2(a)). If the sunspot is good, the government will borrow more (progressing to point B), at which point another bad sunspot realization will induce it to jump to a debt level of about 19% with no possibility of retreat.

Panel 5(b) of Figure 5 illustrates the case corresponding to the second column of Table 1, which we refer to as "exposed." As the government accumulates debt from zero, none of the lower segments of the policy functions actually cross the 45 degree line. This means that regardless of the realization of the Calvo sunspot, the government will end up jumping to a level of debt of around 18%, which is the region of the Cole-Kehoe-type crisis zone. As a result, while the average bond spread is high, it does not exhibit any volatility over time.

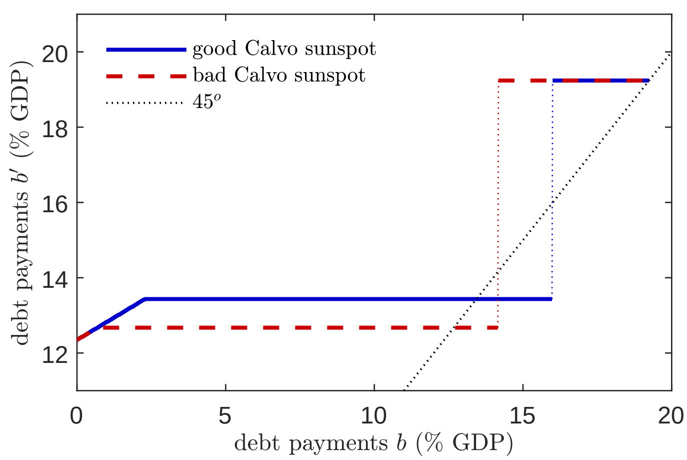
Finally, Panel 5(c) of Figure 5 illustrates the case corresponding to the last column of Table 1, which we refer to as "no default." In this variant, the interval of interest rate multiplicity spans the range of debt where the policy functions are below the 45 degree line and, as a result, the government pulls back. The points of intersection of the policy functions with the 45 degree line lie completely outside of the region of multiplicity, and the government never enters the Cole-Kehoe crisis zone and never defaults. Consequently, the equilibrium bond spread is always zero.



(a) Baseline



(b) Exposed



(c) No default

Figure 5: Optimal borrowing and equilibrium interest rate in the combined model