American Economic Journal: Macroeconomics 2023, 15(1): 106–134
https://doi.org/10.1257/mac.20190189

Learning about Debt Crises†

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The European debt crisis presents a challenge to our understanding of the relationship between government bond yields and economic fundamentals. I argue that information frictions are an important missing element and support that claim with evidence on the evolution of GDP forecast errors after 2008. I build a quantitative model of sovereign default where output features rare disasters and agents learn about their realizations. Debt crises coincide with economic depressions and develop gradually while markets update their expectations about future income. Calibrated to the Portuguese economy, the model replicates the comovement of bond spreads and output before and after 2008. (*JEL* E23, E27, E32, E43, F34, H63)

The recent debt crisis in Europe has reopened the discussion about which factors put governments at risk of sovereign default. The weak correlation between interest rates on public debt and economic fundamentals of the southern European countries challenges the theoretical links established by a large body of research prior to 2008. This new evidence has led some researchers to revisit the hypothesis of self-fulfilling debt crises (Aguiar et al. 2022). Other economists, motivated by the same observations, have argued that the European episode was driven by external factors, such as intra-EU politics (Brunnermeier, James, and Landau 2016). In this paper I propose a quantitative model of the European debt crisis based on idiosyncratic income fluctuations that feature disaster risk and information frictions.

Figure 1 reviews the comovement between government interest rates and external debt, and their relationship with economic fundamentals. The bond spreads[1] were negligible since the introduction of the euro, regardless of current economic performance. At the outset of the Great Recession, peripheral EU countries were hit by negative income shocks in the range between two to three standard deviations below their mean. Yet despite the widespread expansion of external debt

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† Go to https://doi.org/10.1257/mac.20190189 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

[1] The bond spread is defined as the difference between the annual interest rates paid by the given country's government bonds and a risk-free asset, in this case the German long-term government bonds.

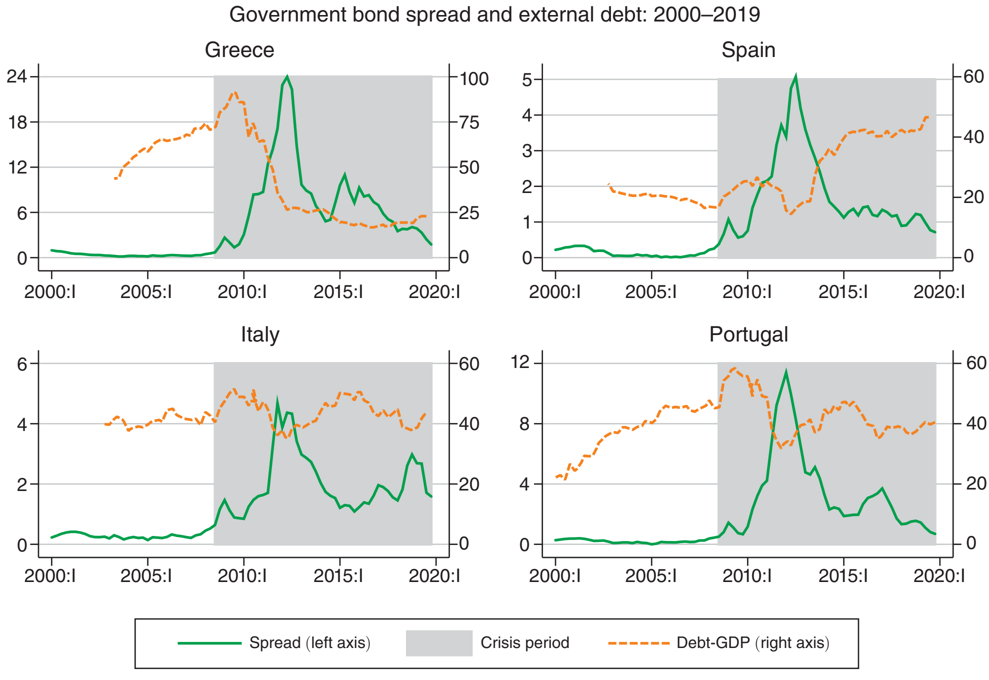

Figure 1. Government Bond Spreads and External Debt of the Peripheral European Economies

Notes: The bond spreads are in percentage points. The debt series represent the external debt securities of the general government and are expressed as a fraction of annualized GDP. The data start at different points in time for different countries. The shaded area starts from 2008:III and represents the beginning of the crisis.

Sources: The bond spreads are acquired from OECD (2020). The data come from the World Bank's (2020) Quarterly External Debt Statistics.

levels in 2009, government bond spreads temporarily rose above 1.5–2.5 percent in 2009:I, only to fall back below 1–2 percent by the second half of 2009.[2] The real stress did not start to build until mid-2010, when Greece experienced a sharp increase in borrowing costs, followed by other countries over the next two years. This slow evolution of the European crisis seems puzzling from the point of view of basic default models where interest rates are sensitive to income shocks and tend to comove positively with debt issuances.

To address this puzzle, I build on two observations about the European debt crisis. First, the experience of southern European economies in years 2008–2014 is much more than just a regular recession. The peak-to-trough decline in quarterly real GDP ranges from around 10 percent for Italy, Portugal, and Spain to 30 percent for Greece. As such, the experiences of these countries qualify for what Barro (2006) refers to as a rare disaster or what Kehoe and Prescott (2002) define as a great depression episode. Second, expectations about future income shocks evolved gradually among financial institutions in years 2008–2014, which I document using

[2] Online Appendix A also shows that sovereign ratings assigned to these countries by the leading credit agencies closely mirrored the dynamics of bond spreads, with major downgrades only starting in 2010–2011.

the real-time GDP forecast data. In particular, prior to 2008, the average forecast bias is at a similar level across all analyzed countries and forecasting agencies, below 1 percent of 2010 real GDP. Then, errors increase to 1.5–6 percent between 2008 and 2011, in all cases *overpredicting* future GDP growth. This indicates that negative output shocks at the time were perceived as a fairly typical recession, corresponding in size and persistence to Europe's postwar business cycle. Finally, in years 2012–2014 the forecasts become much more precise, with average bias falling back below 1 percent and often changing sign (i.e., underestimating future GDP). This increase in pessimism about the countries' economic outlook coincided with the dramatic spikes in interest rates on European governments' bonds.

Motivated by these two observations, I develop an otherwise standard model of sovereign debt that captures the two elements described above in a simple way. Starting with a long-term debt model as in Chatterjee and Eyigungor (2012) or Hatchondo and Martinez (2009), I first introduce a regime-switching income process where a "rare disaster" is represented by a large and negative shift in the long-run mean income of the economy. Then, I assume agents have incomplete information about the underlying switches between the regimes and learn about them over time in Bayesian fashion. As is typical for such a setting, bond yields carry a default premium that varies with the amount of outstanding debt and the expected fluctuations in future GDP. Crucially for my model, spikes in default risk coincide with rare disaster episodes rather than the recurring business cycle downturns, as it is the case for emerging markets. In normal times, which can last for decades, there is little concern about a sovereign debt crisis in the foreseeable future, and, consequently, bond prices carry a negligible default premium. When a rare disaster occurs, income is set on a downward trajectory; however, agents are not aware of this immediately. In other words they cannot tell if the shocks they are observing are temporary or permanent in nature. In the presence of long-term debt, this information friction relieves the upward pressure on interest rates because investors remain optimistic about the economy's long-run outlook. Over time, as income continues to decline, agents increasingly recognize the looming disaster and revise their forecasts. The result is a sudden, sharp spike in default risk that follows long periods of relative calmness in the bond markets.

I use Portugal as a quantitative case study and calibrate the parameters of the regime-switching process using the aforementioned data on real-time GDP forecasts. The calibrated model exhibits interesting behavior in several ways. First, it features a highly volatile bond spread even though average spread is targeted at a low level. This result reflects the fact that the spread tends to be negligible for long periods of time while the good regime is in place, and then it shoots up and remains high when a disaster activates. Second, unlike in an off-the-shelf sovereign default model, my calibration requires a high value of the discount factor, reducing the typical high volatility of consumption or trade balance. This is due to the fact that defaults are generated by the occurrence of rare disasters, while the government behaves countercyclically during "normal times." Third, the government in the model sells bonds at steep discounts, resulting in high equilibrium interest rates, on average reaching 22 percent in the simulated crises. This occurs whenever the belief about an upcoming disaster increases suddenly, while the income level is still high and default is costly. The entire bond price schedule shifts downward, and the government is left with no other choice than to accept very low prices, until either it manages to deleverage or until income is low enough to justify a default.

In an event study of Portugal's debt crisis, I feed in the sequence of GDP data for years 1998–2019. Consistent with the data, the model predicts a negligible spread and slow debt accumulation prior to 2009. Then, the initial adverse income realizations cause an increase in the bond spread to 1.1 percent in the first quarter of 2009, which matches the data and contrasts with the increase to almost 5 percent for the standard, off-the-shelf AR(1) model. This is because the agents are unsure if they are observing temporary shocks or a permanent regime switch. Over the next two years, the belief about the latter converges to certainty, and markets become convinced that the process has switched to a disaster. As a result, in 2012 we observe a delayed jump in the bond spread combined with a sharp reduction in government debt (and, in fact, an eventual sovereign default[3]). The model's predictions for the debt and spread also converge with the data in the postbailout time period from 2016 until the end of 2019. More generally, the Portuguese bond spread closely tracks the (log) of the belief about the disaster realization for the entire period of 2000–2019. Finally, I show that learning is crucial to generate these predictions. In a counterfactual exercise where agents are fully aware of the upcoming disaster by the end of 2008, the bond spread increases in a similar fashion as in the standard AR(1) model, while the government begins a drastic path of debt reduction.

Literature Review.—This paper is closely related to the quantitative sovereign debt literature; in particular it builds on the seminal work of Eaton and Gersovitz (1981) and, more recently, Aguiar and Gopinath (2006) and Arellano (2008). Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) introduce long-duration bonds to these models and show that they are important in accounting for the amounts of debt and average spreads observed in the data.

A recent branch of quantitative default literature investigates the ability of such models to match the volatility of sovereign spreads, with a focus on the European debt crisis. Aguiar et al. (2016) point out that calibrated long-term debt models often deliver a standard deviation of the spread an order of magnitude lower than what it is in the data. To address this issue, Aguiar et al. (2022); Ayres et al. (2019); and Lorenzoni and Werning (2019) revisit models with multiple equilibria to justify why bonds are often sold at large discounts, while Paluszynski and Stefanidis (forthcoming) show that much of the missing spread volatility may be due to the frictions in adjusting government spending. Bocola and Dovis (2019) use the observed maturity choices to identify the rollover risk component of sovereign spreads. Bocola, Bornstein, and Dovis (2019) emphasize the role of domestic debt in generating a high spread volatility relative to its mean. The model in this paper achieves a similar objective of generating volatile and high-peaking spreads through a mechanism of learning about rare disasters.

[3] Recall that Portugal, together with other European countries, received official bailouts in excess of 40 percent of their GDP to prevent them from defaulting, an element not present in my model.

Chatterjee and Eyigungor (2019) present a model of sovereign default with political frictions that shares many similarities with the model in this paper, such as a regime-switching income process. In particular both models obtain volatile bond spreads with a relatively high value of the discount factor. What differentiates them is that their paper is interested in endogenous political turnover in emerging market economies (such as Mexico, Peru, or Turkey), while I focus on the European debt crisis. The main advantage of my paper is that I discipline the key parameters of my model using the data of real-time forecasts, which allows me to capture the slow learning process. This is crucial for generating the correct predictions for the evolution of bond spreads at the outset of the Great Recession in Europe.

On a more general level, this paper is related to two strands of literature in macroeconomics and finance. The first strand introduces rare disasters to otherwise standard macroeconomic models, motivated by Rietz (1988) or Barro (2006). Several papers have recently used this concept in the context of sovereign debt.[4] The second strand incorporates learning about unobserved economic conditions in macroeconomics. Boz and Mendoza (2014) present a model where households learn about the probability of switching between credit cycles to produce a boom-bust cycle like the one observed around 2008 in the United States. Boz, Daude, and Durdu (2011) show that learning about the permanent versus transitory nature of shocks can explain some of the observed differences in volatility between developed and emerging economies. My paper combines these two strands of literature and shows how the data on real time forecasts can be used to estimate the parameters of a rare disaster and to improve the model's predictions.

The remainder of this paper is structured as follows. Section I describes the motivating evidence regarding the European debt crisis. Section II introduces the main model. Section III calibrates the model and uses it to analyze the European debt crisis and contrast the results with those obtained using a benchmark version of the model. Section IV concludes.

I. Empirical Motivation

In this section I document the two pieces of evidence that serve as the main motivation for the model in Section II, namely the depth of the output declines and the evolution of forecast errors in years 2008–2016.

A. *Depth of GDP Drops*

In order to highlight the magnitude of the decline in economic activity among the peripheral European countries, Table 1 lists the largest peak-to-trough drops since 2007. The numbers provided refer to the drops in real GDP both at face value and in relation to a 2 percent trend. Notice that the former ranges from almost 10 percent for Spain, Italy, and Portugal up to over 30 percent in the case of Greece. The cutoff size for a contraction in face value that defines a rare disaster in Barro

[4] See, for example, Ayres et al. (2019); Aguiar and Amador (2020); or Rebelo, Wang, and Yang (2021).

Table 1—Peak-to-Trough GDP Drops among the Peripheral European Economies

Country	Largest decline (in percent) *Face value*	Largest decline (in percent) *Detrended*	**Quarters**
Greece	30.6	39.9	23
Spain	9.1	19.0	21
Italy	9.4	21.4	19
Portugal	9.7	19.4	21

Note: This table shows the largest recorded declines in real GDP in the period 2007–2014, measured at face value and detrended with 2 percent trend as in Kehoe and Prescott (2002).

(2006) is 15 percent. He emphasizes, however, that using an alternative threshold of 10 percent delivers similar results in terms of solving the equity premium puzzle. On the other hand, Kehoe and Prescott (2002) define the "great depression" episode as a sustained negative deviation of at least 20 percent in the GDP level net of the 2 percent annual trend growth. Table 1 indicates that all the economies of interest are around this threshold or clearly above (Greece); the decline is also sustained in time.[5]

B. *Market Expectations during the Recession*

As a second piece of evidence, I investigate the paths of forecasts about GDP growth in real time. The distribution of future income shocks is a crucial element driving interest rate fluctuations in sovereign default models and thus deserves particular attention.

Figure 2 presents the plot of real GDP over time for the four European countries, along with the GDP forecasts published every year by OECD.[6] As can be noticed for the period prior to 2008, while the European economies are still growing along a stable trend, the observed forecast errors are small (with some overshooting for Italy and Portugal, whose economies experienced a slowdown in the early 2000s). When the financial crisis breaks out, the forecasts are still fairly optimistic, predicting a recovery in years 2008–2010. Over time however, as the GDP continues to plunge, we also observe that the forecasts become flatter, indicating that the markets have realized the recovery of output cannot be expected in the short and medium term. From 2012 on, the forecasts essentially line up again with the subsequently realized data for all of the depicted economies. This is also the time when the European bond

[5] Using the countries' individual trends rather than the common 2 percent growth rate would make this conclusion similar or even starker.

[6] For illustration, in this figure I use the two-year-ahead forecasts of real GDP growth from the fall issues of OECD (2020)'s Economic Outlook (the spring version only provides one-year-ahead forecasts). In what follows I also present the forecast errors from other institutions, public and private alike.

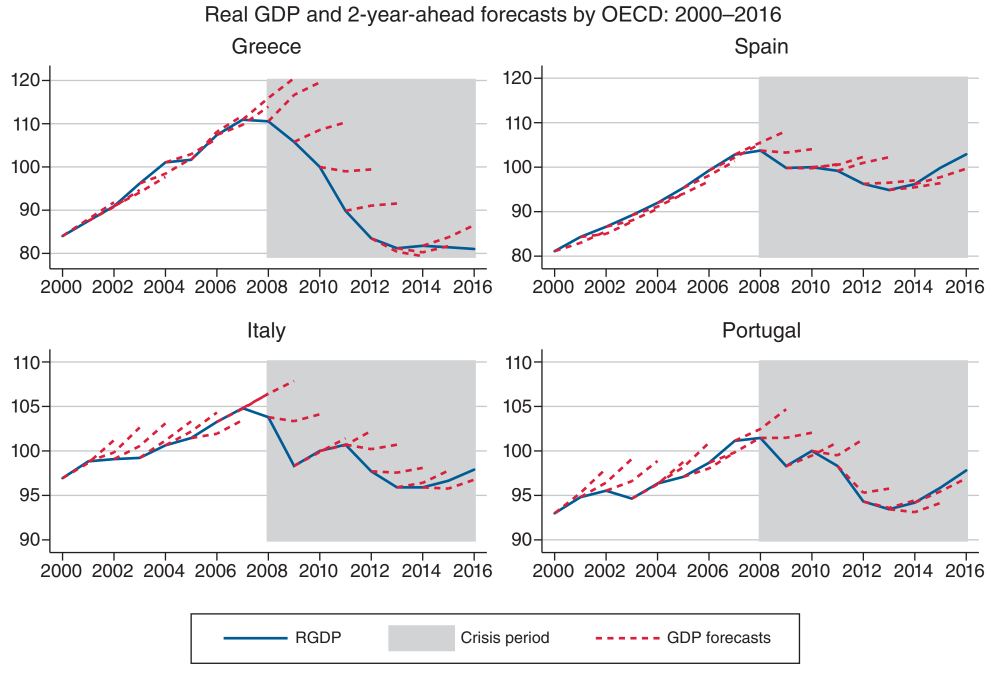

Figure 2. Forecast and Actual Real GDP for the Peripheral European Economies

Notes: The GDP series are annual and expressed in constant prices; their values are normalized such that the observation for 2010 equals 100. The red dotted lines represent one- and two-year-ahead forecasts published by the OECD (2020) Economic Outlook (fall edition of each year) and start in the year when each of them is made.

markets undergo major turbulences that feature surging interest rates, sovereign bailouts, and drastic reductions in debt levels, as documented in Figure 1.[7]

To analyze the forecast errors around the Great Recession more systematically, I acquire real-time predictions from three organizations: OECD, IMF, and the European Commission. The historical forecasts of these institutions are available publicly and released every year in two vintages, spring and fall (roughly corresponding to May and November, respectively, so they are based on the knowledge of the data for first and third quarter of that year). In addition I obtain the data from Consensus Economics, a survey of forecasters from private banks, government agencies, think tanks, and research centers. The queries are collected monthly and ask about a number of macroeconomic indicators, in particular the real GDP growth. A consensus forecast is defined as the average prediction across all participants of the survey. Broad literature documents that consensus forecasts do not suffer from many of the biases that may affect private and public institutions alike and have traditionally performed better than any individual forecaster over the long run (see, e.g., Batchelor 2001 and Cimadomo, Claeys, and Poplawski-Ribeiro 2016). In

[7] Online Appendix A shows that an analogous learning process was absent around Argentina's episode in 2001, the most prominent case study of an emerging market sovereign default.

Table 2—Average Bias in Real-Time Historical Forecasts for Different Time Frames

Average bias	OECD	IMF	EC	CE
Panel A. Pre-recession sample: 2000–2007				
Greece	−0.37	0.05	−0.35	0.00
Spain	0.10	0.03	0.07	0.16
Italy	−0.97	−1.18	−1.04	−0.97
Portugal	−0.97	−0.94	−0.75	−0.96
Panel B. Recession—First stage: 2008–2011				
Greece	−6.53	−6.85	−6.91	−6.73
Spain	−2.19	−2.42	−2.32	−2.33
Italy	−2.19	−1.98	−2.21	−2.05
Portugal	−1.53	−1.75	−1.31	−1.73
Panel C. Recession—Second stage: 2012–2014				
Greece	−0.21	−1.09	−1.05	−0.16
Spain	0.90	0.83	0.65	0.88
Italy	−0.51	−0.70	−0.85	−0.64
Portugal	0.42	−0.07	−0.03	0.59

Notes: The table presents average errors of one-year-ahead forecasts of real GDP level. The bias is expressed as a percentage of the 2010 level of real GDP for each of the four countries. A positive value for the bias indicates that the forecasts underestimate the actual values, while a negative bias indicates that the forecasts overestimate them. All forecasts come in two vintages, spring and fall, which I use jointly. The number of forecasters participating in Consensus Economics surveys varies over time and across countries, with a minimum of 4 and a maximum of 20 in the entire sample.

Sources: Forecasts are acquired from four sources: OECD (2020); IMF (2020); European Commission (2020); and Consensus Economics (2016).

Table 2 I test this conclusion for the southern European economies and investigate the size and direction of forecast errors.

I use one-year-ahead forecasts from each of the four institutions, combining both vintages in every year.[8] To ensure direct comparability, I consider the May and November issues of the Consensus Economics survey, which roughly coincide in time with the spring and fall reports of the IMF, OECD, and the European Commission. Table 2 reports the average bias in the GDP forecasts for the four countries and four sources of interest during three separate time periods: a pre-recession sample of 2000–2007 and the two stages of the Great Recession, namely 2008–2011 and 2012–2014. The bias is expressed as a percentage of each country's 2010 real GDP level and takes negative values when forecasts *overshoot* the actual realizations. There are several interesting observations about these data. First, prior to 2008 the average bias is of similar size across the forecasting agencies, generally under 1 percent of 2010 real GDP. Second, in years 2008–2011 the errors increase sharply, ranging from 1.5–2.5 percent for Spain, Italy, and Portugal up to 6.5 percent for Greece. Third, in years 2012–2014 the bias drops significantly for all analyzed countries, in most cases below the average level from before 2008 (some of the forecasts, in

[8] While OECD and the European Commission also publish up to two-year-ahead, and IMF up to five-year-ahead, forecasts, the Consensus Economics survey is limited to next-year predictions only.

fact, underpredict the GDP level in that point). Finally, the Consensus Economics forecasts *do not* outperform the international organizations, especially during the first stage of the recession in 2008–2011. Interestingly, consensus forecasts do better prior to 2008, in line with the findings of earlier studies such as Batchelor (2001). This indicates that private sector expectations were likely to exhibit excessive optimism, especially at the beginning of the Great Recession.[9]

Large forecast errors at the height of the crisis became a subject of intense critique and led the OECD to publish a study to evaluate the source of mistakes. In a "Post Mortem" Pain et al. (2014, 6) write,

> GDP growth was overestimated across 2007-12.…The largest errors were made at the height of the financial crisis in 2009 but there were also growth disappointments during the recovery.
>
> The OECD was not alone in finding this period particularly challenging. The profile and magnitude of the errors in the GDP growth projections of other international organisations and consensus forecasts are strikingly similar.

In their ex post reflection, the OECD points to the repeated expectation of a swift recovery as the main source of forecast errors, which suggests that a learning process was taking place. To not appear as the only culprit, the OECD also emphasizes that the overly optimistic forecasts have been common among other influential forecasters associated with international bodies and consensus measures. This claim is confirmed by Figure 2 and Table 2.

II. Model

In this section I present a model of sovereign debt that features an augmented specification of the income process and incomplete information about its realizations.

A. *Economic Environment*

Consider a representative agent small open economy with a benevolent sovereign government that borrows internationally from a large number of competitive lenders. Time is discrete, and there is no production or labor. Instead, the economy faces a stochastic stream of endowment realizations. Markets are incomplete, and the only asset available for trading is the multiperiod, noncontingent bond.

Endowment Process.—Suppose the country's endowment follows an autoregressive regime-switching process. I assume that there are two possible regimes, High

[9] While average bias is a useful measure for evaluating the direction of forecast errors, it does not necessarily present their full magnitude. This is because errors of opposite signs may cancel each other out over time. Moreover, even if errors generally go in the same direction (just like during the Great Recession), the average bias aggregates them linearly, i.e., an error of 2 percent is equivalent to two errors of 1 percent each. As a result, it treats large errors during events like the Great Recession with a similar weight as several small errors combined in the period prior to 2007. To address this issue, online Appendix A presents analogous calculations for the root mean square errors (RMSE). As a nonlinear measure, RMSE punishes infrequent large errors more heavily than frequent small ones. Indeed, the analysis of RMSEs supports all four observations discussed above.

and Low, and each of them is characterized by its own long-run mean. For simplicity, the persistence and variance parameters are assumed to be constant across regimes. Specifically, the evolution of output, detrended with a deterministic long-run mean growth rate,[10] is given by

$$y_t = \mu_j(1-\rho) + \rho y_{t-1} + \eta\epsilon_t, \tag{1}$$

where $\epsilon_t \sim \mathcal{N}(0,1)$ is an i.i.d. random shock and $\rho, \eta, \{\mu_j\}_{j=L,H}$ are parameters of the two regimes. Regimes change according to a Markov process with the transition probability matrix given by

$$\Pi = \begin{bmatrix} \pi_{LL} & 1-\pi_{LL} \\ 1-\pi_{HH} & \pi_{HH} \end{bmatrix}. \tag{2}$$

The specification of a bimodal stochastic process of endowment in formula (1) is nonstandard in the sovereign debt literature.[11] It is motivated by the income pattern of European economies in the recent decade, illustrated in Table 1. Throughout the paper I will consider the two regimes as highly asymmetric, with the low one having the interpretation of a rare disaster or a great depression.

Preferences.—The representative household has preferences given by the expected utility of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{3}$$

where I assume the function $u(\cdot)$ is strictly increasing, concave, and twice continuously differentiable. The discount factor is given by $\beta \in (0,1)$.

Government.—In each period the government chooses a consumption rule and the level of debt holdings to maximize the household's lifetime utility. The only asset available is the long-duration bond. In the spirit of Chatterjee and Eyigungor (2012), I assume that each unit of outstanding bonds matures probabilistically in every period or pays a fixed coupon. The government may save at an international risk-free interest rate. If it decides to borrow, however, the government is not committed to repay the debt next period. Consequently, the bond is priced endogenously by risk-neutral lenders to account for the possibility of default as well as debt dilution in the future. As is commonly assumed in the sovereign debt literature, the government who refuses to honor its obligations faces an exogenous cost of default

[10] Online Appendix B discusses details of the detrending method used in this paper.

[11] Simultaneously with the present paper, a bimodal income process was also used by Chatterjee and Eyigungor (2019) and Ayres et al. (2019). In contrast to these papers, the regimes here are meant to be highly asymmetric, which can be interpreted as "normal times" and "rare disaster." In such a setting learning about the underlying regime has a naturally powerful effect, as I show in Section III, because agents tend to have a strong prior belief against a potential regime switch. I also show a new method of estimating this income process by incorporating real-time forecast data.

and is further excluded from borrowing in the financial markets, with a certain probability of being readmitted in every subsequent period.

Market Clearing.—There is no storage technology, which, under the aforementioned assumptions on the utility function, implies that the endowment is fully divided between current consumption and net borrowing. This market clearing condition is given by

$$c_t = y_t - b_t\left[\delta + (1-\delta)\kappa\right] + q_t\left[b_{t+1} - (1-\delta)b_t\right], \tag{4}$$

where q_t is the price of the debt stock b_{t+1} (to be repaid next period), δ is the rate at which bonds mature every period, and κ is a fixed coupon.

Bond Prices.—International lenders are perfectly competitive and have "deep pockets" in the sense that potentially even large losses do not affect their decisions. In equilibrium the lenders make expected zero profit, and as a result, the bond pricing formula compensates them only for the default risk implied in the government's decisions.

B. *Information Structure*

The two state variables mentioned so far, current bond holdings (b_t) and income (y_t), are standard in sovereign debt literature. In addition this model features another exogenous stochastic variable, $z_t \in \{z_L, z_H\}$, representing the regime (Low or High) in which the economy is currently operating. While all agents know the latest income realization, they have incomplete information about the current regime. Instead of observing it directly, agents form a belief p_t defined as their perceived probability of being in the High regime, formally $p_t \equiv \Pr(z_t = z_H)$. Intuitively, this variable can be thought of as market sentiment about the economy's expected future income path. As I show in Section III, the belief about regime is quantitatively significant and appears to have fluctuated substantially in years 2008–2014.

C. *Timeline*

In every period the timing of events is as follows:

(1) The new regime $z \in \{z_L, z_H\}$ is drawn, with the probability distribution given by equation (2).

(2) The new realization of endowment y is drawn, according to the newly updated regime z and conditional on its level from last period.

(3) Agents observe y and mechanically form a new belief p about the regime, conditional on the previous and current endowment as well as the last period's belief.

(4) Default and redemption decisions take place:

- The government that has recently defaulted on its debt draws a random number to determine whether it can be readmitted to the financial markets.
- The government that has recently been current on its debt decides whether to repay or default this period.

(5) Equilibrium allocations take place:

- If the government defaults, it is excluded from financial markets this period and simply consumes its endowment, subject to a default penalty.
- If the government repays, it chooses the new allocation of bonds b', while the lenders post the bond price $q(b', y, p)$.

D. *Recursive Formulation*

In the following section I formalize the economic environment by stating the problems faced by market participants in recursive form. To begin, define the vector of aggregate state variables that are common knowledge as $\mathbf{s} = (b, y, p)$.

Government.—The government that is current on its debt obligations has the general value function given by

$$v^0(\mathbf{s}) = \max_{d \in \{0,1\}} \left\{ (1-d) v^r(\mathbf{s}) + d v^d(y, p) \right\}. \tag{5}$$

A sovereign who defaults $(d = 1)$ is excluded from international credit markets and has probability θ of being readmitted every subsequent period with zero debt. The assumption that all debt is wiped out upon readmission is not necessary and can be relaxed at the expense of complicating the analysis. The associated default value is

$$\begin{aligned} v^d(y,p) = {} & u(y - h(y)) + \beta \sum_{z \in \{z_L, z_H\}} \sum_{z' \in \{z_L, z_H\}} \Pr(z) \pi(z'|z) \\ & \times \int f_{z'}(y', y) \left[\theta v^0(0, y', p') + (1-\theta) v^d(y', p') \right] dy', \end{aligned} \tag{6}$$

subject to the law of motion for the belief

$$p'(y, p, y') = \frac{\left[p\pi(z_H|z_H) + (1-p)\pi(z_H|z_L) \right] f_{z_H}(y', y)}{\sum_{z' = z_L, z_H} \left[p\pi(z'|z_H) + (1-p)\pi(z'|z_L) \right] f_{z'}(y', y)}. \tag{7}$$

In equation (6), $h(\cdot)$ is a reduced-form representation of the output cost of defaulting;[12] $f_{z'}(y'|y)$ denotes the probability density of transitioning from state y to

[12] Quantitative sovereign debt models typically assume an exogenous punishment in the case of default in order to facilitate calibration of the model to the data. For the specific functional form, see Section IIIC.

state y' given that tomorrow's regime is z'. $\Pr(z_H)$ is equal to p and $\Pr(z_L)$ is $1-p$; $\pi(z'|z)$ is the probability of transitioning from regime z today to z' tomorrow. The next-period belief p', described in equation (7), depends on the current and future income realization as well as the current belief p. It is a simple application of Bayes' rule and takes into account a potential regime switch at the beginning of next period, according to the transition matrix given by equation (2).

The value of the government associated with repayment of debt is given by

$$(8)\quad v^r(\mathbf{s}) \;=\; \max_{c,b'}\left\{u(c) + \beta \sum_{z\in\{z_L,z_H\}} \sum_{z'\in\{z_L,z_H\}} \Pr(z)\pi(z'|z)\int f_{z'}(y',y)v^0(\mathbf{s}')dy'\right\},$$

subject to the law of motion for the belief in formula (7) and

$$(9)\qquad c \;=\; y - b\big[\delta + (1-\delta)\kappa\big] + q(b',y,p)\big[b' - (1-\delta)b\big],$$

where equation (9) is the budget constraint.

Having characterized the two value functions of the government, it is straightforward to derive the optimal default policy as a function of today's state variables

$$(10)\qquad d(\mathbf{s}) \;=\; \begin{cases} 1, & \text{if } v^d(y,p) \;>\; v^r(\mathbf{s}); \\ 0, & \text{if } v^d(y,p) \;\le\; v^r(\mathbf{s}). \end{cases}$$

International Lenders.—Every period the lenders only observe (b,y) and share a common market belief p. Although they do not see the current regime z, they know its distribution and independently update their belief about it, as described by the law of motion in formula (7). The denominator in those equations is always greater than zero, and the resulting next-period belief p' is strictly interior on the interval $(0,1)$.

As is common in the quantitative models of sovereign debt, lenders are competitive and risk-neutral by assumption. The resulting equilibrium bond price is such that they make zero profit in expectation (according to their imperfect information). The bond price function is

$$(11)\quad q(b',y,p) \;=\; \frac{1}{1+r^*}\left\{\sum_{z\in\{z_L,z_H\}} \sum_{z'\in\{z_L,z_H\}} \Pr(z)\pi(z'|z)\int f_{z'}(y',y)\big(1-d(\mathbf{s}')\big)\right.$$
$$\left.\times\;\Big[\delta + (1-\delta)\Big(\kappa + q\big(g(\mathbf{s}'),y',p'\big)\Big)\Big]dy'\right\},$$

where $\mathbf{s}' \;=\; \big(b',y',p'(y,p,y')\big)$; $d(\cdot)$ and $g(\cdot)$ are the government's optimal decisions with respect to default and new debt, respectively; and r^* is the risk-free rate of interest.

Concluding this section, Definition 1 introduces the standard concept of a Markov Perfect Bayesian Equilibrium. In this equilibrium the posterior beliefs of agents must be specified at all states and for all strategies of other players (including those involving off-equilibrium actions). The agents' best responses must belong to the set of stationary Markov strategies.

DEFINITION 1: *A Markov Perfect Bayesian Equilibrium for this economy consists of the government value functions* $v^r(\mathbf{s})$, $v^d(y,p)$ *and policy functions* $c(\mathbf{s})$, $b'(\mathbf{s})$, $d(\mathbf{s})$ *and the bond price schedule* $q(b',y,p)$ *such that*

(*i*) *Policy function d solves the government's default-repayment problem* (5).

(*ii*) *Policy functions* $\{c,b'\}$ *solve the government's consumption-saving problem in* (8).

(*iii*) *Bond price function q is such that the lenders make zero expected profit* (*subject to their imperfect beliefs*).

III. Quantitative Analysis

In this section I calibrate the model to Portuguese data and discuss its mechanics. I then present the simulated behavior of the model and use it to study the European debt crisis.

A. *Data*

I use data from the Portuguese economy as a case study for the theory developed in this paper. The model could also be calibrated to other European economies discussed in Section I. The Portuguese episode is the most clear-cut case, however, because it does not coincide with other major economic events, such as a banking crisis (like the one that occurred in Ireland) or Mario Draghi's "whatever it takes" speech and the introduction of the Outright Monetary Transactions (OMT) program in the summer of 2012, at the peak of the debt crises in Italy and Spain. Portugal is also a particularly relevant laboratory for this type of sovereign default model, as it easily satisfies its core assumptions; i.e., it is arguably a small open economy, and the vast majority of its debt securities were held externally (Andritzky 2012). While in principle Greece is also a plausible candidate, the validity of its macroeconomic data is questionable. Nonetheless, online Appendix B extends the main estimation of the regime-switching income process to other countries mentioned in the introduction and discusses the usefulness of this theory in explaining these cases.

Quarterly data for real GDP are taken from OECD (2020) and cover the period 1960:I–2019:IV. Consumption, trade balance, and interest rates on long-term government bonds, also from OECD (2020), span the time frame of 1998:I–2019:IV. Government debt data are acquired from World Bank (2020)'s Quarterly External Debt Statistics (I use debt securities only).

The identification strategy for the model's parameters is in line with the general approach in the literature. In what follows I first estimate the income process using both historical GDP data and the real-time GDP forecast data introduced in Section IB. Then, I select the remaining structural parameters of the model partly from the literature and partly to match certain general characteristics of the Portuguese economy.

B. *Estimation of the Income Process*

I proceed to calibrate equation (1) in two steps. First, I fix the probabilities of switching into, and out of, the rare disaster regime based on recent historical experience. By all accounts, the recession in southern European countries has been the worst since the Great Depression.[13] This gives us roughly 60 years of high regime duration.[14] On the other hand, the Great Depression lasted for about ten years, which I use to pin down the expected low regime duration. The resulting probabilities of staying in the high and low regimes are therefore 0.996 and 0.975, respectively. The exact numbers behind these probabilities are not crucial for the results because, given their predetermined values, I use two other data sources below to discipline the remaining parameters of the income process. What matters, however, is to capture the right order of magnitude—intuitively, the low regime ought to be rare and severe enough so that it clearly stands out from a regular economic contraction.[15]

In the second step I normalize the high-regime mean μ_H to zero and employ a variant of the Expectation-Maximization algorithm (Hamilton 1990) to estimate the remaining coefficients. Importantly, to capture the slow learning process documented in Section IB, I jointly use two data sources—the historical GDP series for years 1960:I–2019:IV[16] and the real-time GDP forecasts for years 1993–2014.[17] Online Appendix B discusses the details of my estimation technique. Table 3 summarizes the calibrated parameters of the income process.

It should also be emphasized that the estimated regime-switching model in Table 3 provides a better fit to Portuguese data than a single-mean AR(1) process. To show this, I estimate the detrended AR(1) process for years 1960–2011 to be 0.947 and 0.011, respectively.[18] Then, the likelihood ratio test statistic is 22 and

[13] While Portugal was not heavily impacted by the Great Depression itself, it subsequently suffered during the 1934–1936 civil war in Spain, with a peak-to-trough decline in real GDP of 12.5 percent. Following the 1930s, the 2008–2014 episode is by far the most severe contraction in Portuguese economic history. It comes close to satisfying the defining criteria of a great depression established by Kehoe and Prescott (2002). Also Reis (2013) compares the Portuguese episode to the US Great Depression and Japan's lost decade.

[14] I ignore the Second World War in my calculations, as the model is not designed to account for such events.

[15] This type of approach to calibration is common for models with disaster risk. For example, pooling 60 episodes in 35 countries, Barro (2006) sets the probability of entering a disaster event at 1.7 percent annually, almost the same as the number I use. More recently, Coibion, Gorodnichenko, and Wieland (2012) take a similar approach to approximate the frequency of interest rates in the US hitting the zero lower bound, which occurred for the first time since the Second World War.

[16] Because of the growth trend changes for European economies over this relatively long sample, I follow Bai and Perron (1998) to identify statistically significant structural breaks in the growth rate of the Portuguese economy. Two breakpoints are detected, at 1974:II and 1999:IV, which is intuitive, as they coincide with the democratic revolution in Portugal and accession to the eurozone, respectively. The estimated quarterly trend growth rates for the three time windows are 1.6 percent, 0.8 percent, and 0.4 percent. Including all of the available information since 1960 is important to capture the full scope of the variance of GDP while the high regime is in place, i.e., during "normal times." This is an issue especially for the European countries whose GDP data exhibit very little variance in years 1999–2008 alone. See online Appendix B for more details on the detrending.

[17] Specifically, I use five-year-ahead projections published by the IMF, given that they provide the longest time series and the longest forecasting horizon. I do not include the post-2014 forecasts in this estimation to avoid capturing the reversal of pessimism documented in Section IB. See also Paluszynski (2021), which uses long-term forecast data to calibrate the parameters of a stochastic process for the risk-free interest rate.

[18] Once again, I use a broken linear trend with two statistically significant breakpoints detected using the Bai and Perron (1998) test at 1974:II and 1999:IV. Note that using the GDP data all the way until 2019 produces similar estimates and mostly does not affect the results of the paper. Its interpretation is problematic, however, because it implies that the slope of the trend has been essentially zero since 1999, and thus year 2008 appears to be the peak of a historic boom for the Portuguese economy.

Table 3—Parameters of the Regime-Switching Endowment Process

				Transition prob.	
Regime	Mean μ	Persistence ρ	SD η	Low	High
Low	−0.291	0.970	0.010	0.975	0.025
High	0.000	0.970	0.010	0.004	0.996

has approximately a χ^2 distribution with three degrees of freedom (the number of additional parameters in the extended model), resulting in a p-value much smaller than 0.01. This suggests that we can reject the null model (single-regime AR(1)) at virtually all levels of significance.

Concluding this section, it is natural to ask about the results of a more standard estimation technique that does not use the real-time forecasts as an additional data source. Such an approach would yield the following estimates for the sample from 1960 to 2011: $\rho = 0.95$, $\eta = 0.01$, $\mu_H = 0.03$, $\mu_L = -0.32$, $\pi_{HH} = 0.96$, $\pi_{LL} = 0.71$. Notice that now the high regime mean is no longer normalized to zero, while the low regime has the unconditional mean similar to the one reported in Table 3. Crucially, switching is much more likely under this specification, and the regimes do not last very long. We detect several instances of the low regime over time, in particular around 1969, 1974, 1983, 1992, 2008–2009, and 2011–2014. Because of such frequent switching, the learning process about an underlying switch is fast and does not align well with the evidence from real-time forecasts presented in Section IB.

C. *Functional Forms and Calibration*

To select the remaining structural elements of the model, I follow the general trends in the literature by fixing the value of some noncontroversial parameters, and I use a moment-matching exercise to pin down the more problematic ones. The representative household's utility is a CRRA function of the form $u(c) = c^{1-\sigma}/(1-\sigma)$, with risk aversion parameter set at the standard level of 2. The risk-free interest rate is set equal to 1 percent (quarterly value), and the probability of reentry after default is fixed at 0.049, following Cruces and Trebesch (2013), who find that the average time to reenter the credit market was 5.1 years in 1970–2010. Using OECD data, I find that the average maturity of Portugal's debt in years 1996–2010 was 4.73 years, which translates into an average quarterly maturity rate of 0.053. The coupon payment is set to 1.25 percent following Salomao (2017), which implies an annual coupon of 5 percent .

The output cost of default is parameterized as $h(y) = \min\{y, \hat{y}\}$ following Arellano (2008). The parameter $\hat{y}$ is calibrated jointly with the discount factor β using the simulated method of moments. The economy's income path in years 1998–2019 is simulated 10,000 times, starting from the actual GDP and debt levels observed in 1998:I and under the assumption that the regime switches from High

Table 4—Calibration of Structural Parameters of the Model

Symbol	Meaning	Learning	AR(1)	Source
σ	Risk aversion	2	2	Literature
r^*	Risk-free rate	0.01	0.01	Literature
θ	Reentry probability	0.049	0.049	Literature
δ	Probability of maturing	0.053	0.053	Data
κ	Coupon payment (in %)	1.250	1.250	Data
$\hat{y}$	Default cost par.	0.793	0.919	Calibration
β	Discount factor	0.988	0.980	Calibration
Calibration targets		Learning	AR(1)	Data
E(debt/GDP)		38.59	38.58	38.58
E(spread)		1.75	1.75	1.75

Notes: Targeted moments are given in percentage points. Simulations are repeated 10,000 times for the period of 1998–2019.

to Low in 2008:III.[19] The idea behind the identification strategy is to match certain general characteristics of the Portuguese experience during that time period. To this end, I use information from two standard moments of Portugal's economy in years 1998–2019: average ratio of external debt securities to GDP of 38.6 percent[20] and average 5-year bond spread of 1.75 percent.[21] The former is naturally an important piece of information to identify the relative impatience of the government and the punishment for default. The latter, in a model with risk-neutral lenders and zero recovery rate, simply reflects the government's average default probability. The target of 1.75 percent is thus reasonable given all the independent evidence on Portuguese default history.[22] Table 4 presents a summary of the parameter values that provide the closest match to the empirical moments. The model achieves an exact match in terms of both targeted moments. The calibration procedure results in a discount factor β of 0.988 and the default penalty parameter $\hat{y}$ of 0.793.

Finally, in order to make a meaningful comparison with a literature benchmark, I also calibrate a "standard" sovereign debt model with long-term debt, similar to Chatterjee and Eyigungor (2012) or Hatchondo and Martinez (2009), which uses a simple AR(1) specification of the income process. As mentioned before, the estimated persistence and variance parameters are 0.947 and 0.011, respectively. Calibration of the structural parameters closely follows the strategy described above and is summarized in the AR(1) column of Table 4. It is important to emphasize that

[19] An alternative would be to simulate the economy over many years and calibrate to its ergodic (long-run) business cycle statistics outside of default, following Chatterjee and Eyigungor (2012). However, as it will become clear from Table 5, describing the simulation results, years 1998–2019 were a highly nonstationary period for the European economies, including slow accumulation of debt toward a steady-state level.

[20] Because the model does not include postdefault renegotiation, I follow Chatterjee and Eyigungor (2012) to calibrate only the true "unsecured" portion of the debt. While Portugal in the end did not default and it is difficult to know how much of its debt was in fact unsecured, the best guess is 0.535, the haircut rate in the case of the Greek default of 2012.

[21] I use a 5-year spread, rather than 10-year as in the introduction, because Portugal's average debt maturity is 4.73 years. The data on 5-year spread are acquired from Bloomberg L.P. (2017).

[22] Reinhart and Rogoff (2009) identify 4 sovereign defaults in Portugal's history since 1800, while Standard and Poor's (2014) identify 3, implying an annual long-run probability of 1.5−2 percent.

this benchmark is not meant to claim that no model based on an AR(1) process can produce relevant predictions for the European debt crisis. Indeed, several papers, such as Bocola and Dovis (2019) or Salomao (2017), have managed to do so. The benchmark thus refers to an "off-the-shelf" model following a routine calibration approach.

An important conclusion from the moment-matching exercise is that it produces significantly different values of the discount factor in the model with disaster risk and learning, relative to the literature benchmark. In particular in the "standard" model with a simple AR(1) process, a value of β around 0.98 is needed to simultaneously generate high debt and defaults occurring with a desired frequency. On the other hand, in the model with learning about disasters, the same targets are achieved with a discount rate of just under 0.99. This is because defaults here occur predominantly under the circumstances of a rare disaster rather than due to myopic behavior of the borrower. As a result, the model features a government that mostly uses debt for consumption-smoothing purposes but may occasionally default should the output collapse in a Great Depression–like fashion. The following two sections illustrate the behavior of the government in this model in more detail.

D. *Characterization of the Equilibrium*

In the following section I first characterize some of the key properties of the equilibrium and then show how the model's simulated behavior compares with actual data. The model is solved numerically by value function iteration using a continuous choice of next-period debt and cubic spline interpolation (Habermann and Kindermann 2007) to evaluate off-grid points, similarly as described in Hatchondo, Martinez, and Sapriza (2010). Expectations are approximated using Gaussian quadrature with 51 nodes and off-grid points for income, and beliefs are linearly interpolated. I use 41 points for the grid of assets, income, and the belief.

Model Mechanics.—To understand how the model works, it is instructive to examine how the government's optimal decisions change with respect to state variables. Figure 3 shows the default and debt policies for different levels of prior belief. On the left-hand-side panel, any combination of current debt and income above the line corresponding to some belief p indicates repayment, while a combination below the line indicates default. Not surprisingly, higher belief about being in the good regime induces the government to default in a smaller number of states. This relationship is strictly monotonic in the level of prior belief (but not necessarily linear). The right-hand-side panel of Figure 3 shows that higher prior beliefs induce the government to borrow more. In this model agents are impatient and would rather consume today than tomorrow. When making their debt decisions, however, they need to weigh their impatience against the expected income level in the future. A higher chance of being in economic depression next period implies that the government must restrict its consumption today and reduce foreign debt, in order to decrease the probability of defaulting tomorrow and to secure a high bond price today. Consequently, higher market belief has a strictly monotonic, increasing effect on the optimal debt level.

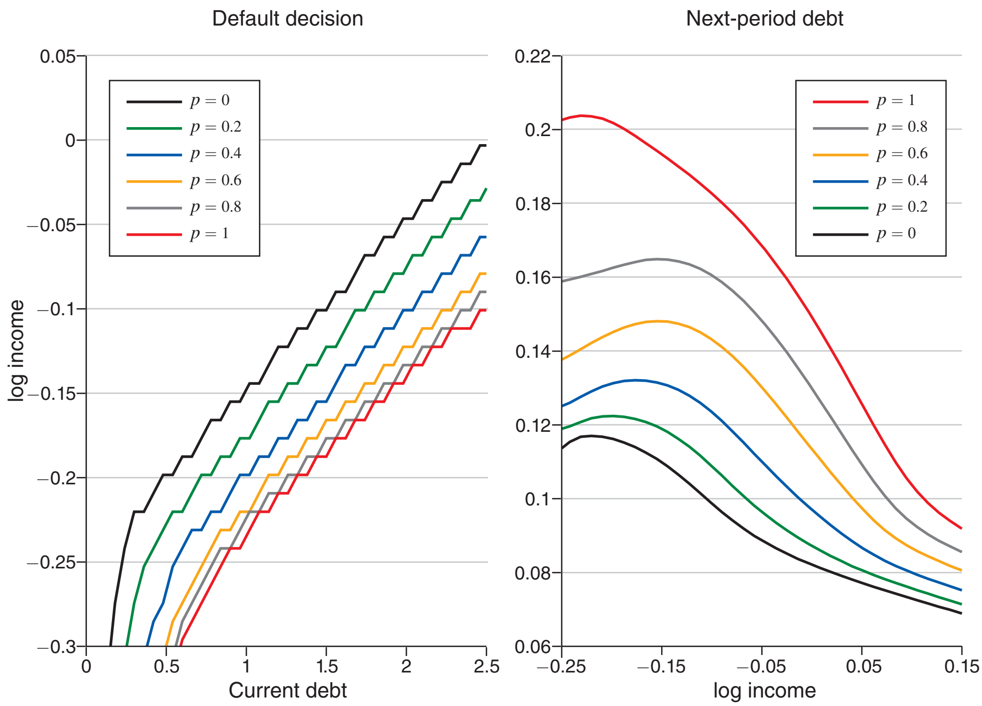

Figure 3. Default Sets and Bond Price Policy Functions for Different Beliefs

Notice furthermore that policy functions are generally decreasing in income, which implies a consumption-smoothing behavior during normal times. It is only when income falls low enough, to a Great Depression–like level, that the policy functions bend over and take an increasing shape, more common for this class of models, resulting in a procyclical fiscal policy.

Figure 4 plots government bond prices as functions of the next-period debt choice, at several different levels of the belief.[23] The information about current regime is important in determining future default risk and leads to large differences in the offered bond prices. The highest (red) line represents the bond price schedule when markets are fully convinced the economy is in the high regime. As a result, the government is able to secure an almost maximum price for its bonds, regardless of its choice of next-period debt (within reasonable bounds). By contrast, the lowest (black) line represents the schedule if the markets believe the economy is currently in a depression. Because default risk is much higher in such circumstances, the government is offered very low prices for its debt. Finally, the schedules in the midrange are increasing monotonically as the belief of being in the high regime rises.

Business Cycle Statistics.—In the next step I analyze the model's behavior in simulations. As discussed in Section IIIC, the model exhibits notably different

[23] This graph does not say anything about optimality of different debt choices; it merely depicts the possible price schedules.

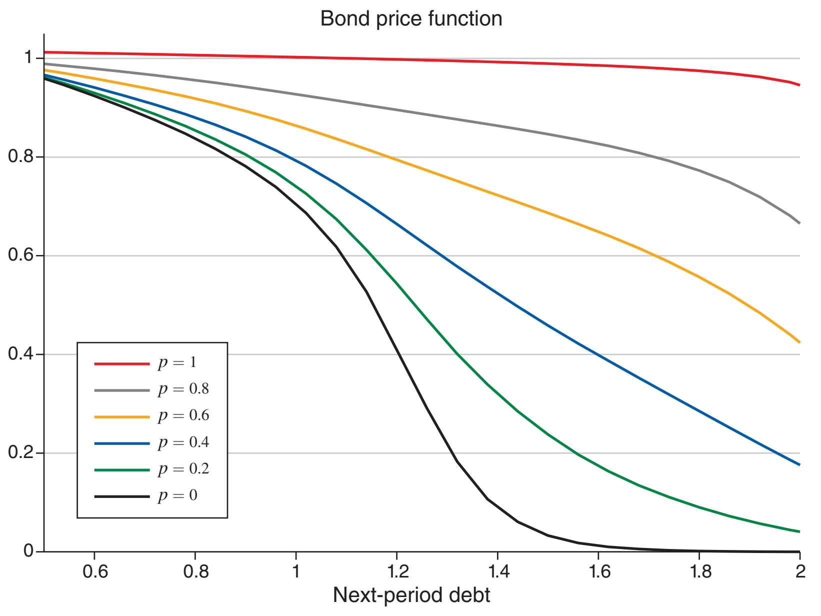

Figure 4. Bond Price as Function of Next-Period Debt for Different Beliefs

Table 5—Simulated Behavior of the Model

		Learning		AR(1)	
Statistic	**Data**	*Ergodic*	*Conditional*	*Ergodic*	*Conditional*
$E(s)$	1.75	1.12	1.75	1.93	1.75
$\text{std}(s)$	3.06	3.90	3.63	1.83	1.42
$\text{std}(c)/\text{std}(y)$	0.98	0.98	1.17	1.22	1.43
$\text{std}(tb)/\text{std}(y)$	0.55	0.26	0.39	0.35	0.78
$\text{corr}(y, c)$	0.98	0.97	0.95	0.97	0.86
$\text{corr}(y, tb)$	−0.96	0.21	−0.29	−0.52	−0.25
$\text{corr}(y, s)$	−0.42	−0.38	−0.71	−0.67	−0.69
$\text{corr}(s, tb)$	0.39	0.38	0.55	0.72	0.66
$E(debt/y)$	38.58	62.16	38.59	43.32	38.58

Notes: Moments for the bond spread (annual) and debt-to-GDP ratio are given in percentage points. Ergodic (long-run) simulations extend to 10,000 quarters and are repeated 10,000 times, following closely Chatterjee and Eyigungor (2012). Conditional (short-run) simulations mimic the period of 1998–2019 (88 quarters) and are repeated 10,000 times starting from the actual levels of debt and GDP observed in 1998:I. Each short-run sample is constructed such that (i) the series start from the actual 1998:I debt and income levels and (ii) the regime switches from good to bad in 2008:III. Consumption data are detrended using the common GDP trend.

behavior in the long run versus the short samples that aim at mimicking the period of 1998–2019 for the European economies.

Table 5 presents the simulated business cycle moments in the long- and short-run samples (i.e., ergodic and conditional distribution), along with the empirical ones. It can be noticed that the model simulated in the short run performs closer to actual

Table 6—Bond Spread Moments in the Simulations and the Data

	Bond spreads (in %)		
Country	μ	σ	c_v
Model—AR(1)	1.75	1.42	0.81
Model—Learning	1.75	3.63	2.07
Data—Portugal	1.75	3.06	1.75

data in terms of correlations between the main variables, and the moments of bond spread and debt. In particular the average debt-to-GDP ratio for Portugal in the long run implied by the model is around 62 percent, much higher than in the data for 1998–2019. This is due to the fact that the government is gradually accumulating debt toward its steady-state level for most of the 2000s. Unlike the ergodic distribution, short-run simulations are able to capture this nonstationarity. Related to the policy functions discussed in Figure 3, notice that the government's ergodic behavior implies a countercyclical fiscal policy, with a positive correlation of income and trade balance (tb), and a low volatility of consumption relative to output. These features do not show up in the short-run statistics, however, due to the disproportionate presence of a rare disaster, which makes the government behave more like a classic sovereign defaulter.

Simulated Behavior of Spreads.—Table 6 highlights the most notable difference between the two models. The first two moments of the simulated bond spreads are presented for each model and contrasted with the data. The average spread is a targeted moment, so it is matched in both cases exactly. However, the standard deviation of the spread is not a calibration target, and for the AR(1) model it falls short of the level observed in the data, producing a *coefficient of variation* of the spread smaller than one (reported in the last column). This result confirms the finding of Aguiar et al. (2016), who show that models of this type generally fail to deliver a realistic volatility of the bond spread. By contrast, the model with disaster risk and learning generates a standard deviation that actually exceeds what we observe in the data, resulting in a coefficient of variation above one. Mirroring this result is the fact that the trade balance is less volatile in the disasters model than in the AR(1) model, as evident in Table 5. By adjusting its trade balance more aggressively, the borrower in the standard model is able to target a desired level of bond spread with smaller variance.

The intuition behind the result presented in Table 6 is the following. In the standard "off-the-shelf" AR(1) model with long-term debt, a sovereign default is possible most of the time, within the expected duration of an outstanding bond. Consequently, the spread never falls to zero, although it may on average be much smaller than the ones obtained for emerging market economies, as shown in Chatterjee and Eyigungor (2012) and other studies. On the other hand, in the model with disasters and learning, sovereign defaults occur almost exclusively when the

Table 7—Bond Spread Statistics for European versus Emerging Market Economies

	Bond spreads (in %)		
	μ	σ	c_v
European			
Greece	3.90	5.94	1.52
Spain	0.98	1.33	1.36
Italy	1.05	1.21	1.15
Emerging			
Argentina	10.25	5.58	0.54
Ecuador	16.91	10.72	0.63
Russia	19.41	17.60	0.91

Note: Bond spread moments for European countries are computed with OECD (2020) data covering 1999:I–2014:IV.

Source: The moments for emerging economies are taken from Arellano (2008).

economy has switched to the disaster regime.[24] Hence, most of the time while the economy is doing well and the market-wide belief is close to one, lenders do not fear that default is a possibility in any predictable future. As a result, bond spreads are very close to zero, and the average spread is low even if a debt crisis eventually does occur. Once that happens, the spread shoots up and can attain large values (which I explain in the subsequent paragraphs), resulting in an overall high standard deviation.

Table 7 generalizes this point by documenting the difference in bond spread moments of other peripheral European countries discussed in Section I and emerging market defaulters (which are the examples originally presented by Arellano 2008). As can be noticed, the former tend to have a coefficient of variation of the bond spread above one, implying that spread volatility is high relative to its average. On the other hand, emerging market defaulters tend to have average spreads that exceed their standard deviations significantly, resulting in a coefficient of variation smaller than one. Consequently, as Table 6 shows, the standard AR(1) model seems to be a better description of the debt crisis experienced by an emerging market economy, while the model with disaster risk and learning presented in this paper is a better description of the recent episode experienced by developed European nations.

Another interesting feature of the model is that the bond spreads can take much higher values on the equilibrium path than in the benchmark; i.e., the government sometimes sells bonds at deep discounts. To illustrate this point, Figure 5 compares the distributions of spreads realized in the simulations for the two models. In the model based on a simple AR(1), spreads essentially do not carry any mass for values above 0.2, and the average maximum spread attained in the short-run simulations corresponding to the data sample is 9.2 percent. By contrast, the distribution of

[24] In the long-run simulations summarized in Table 5, over 94 percent of all defaults occur in the low regime, and the average value of the belief at default is 0.06.

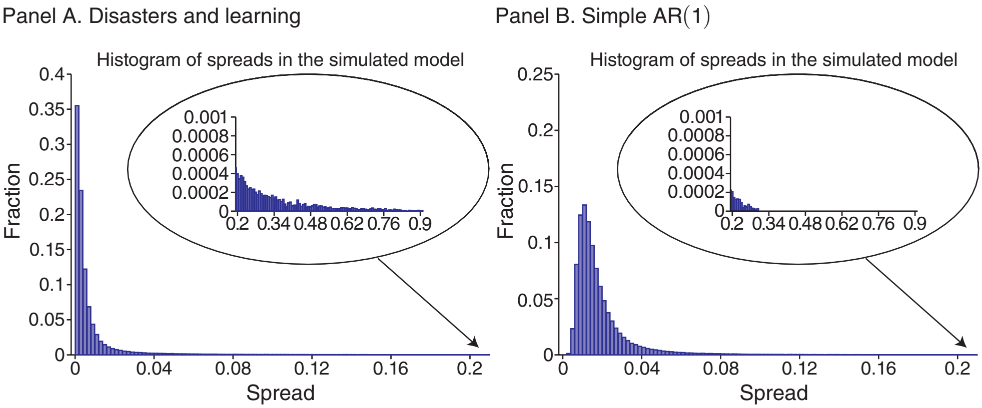

Figure 5. Histogram of Spreads in Simulated Models

spreads in the model with disasters and learning features a long upper tail extending all the way to 100 percent, with the average maximum spread in the conditional distribution of 21 percent. The intuition behind this result is straightforward. In the former the government targets a certain level of spread and only tolerates limited upward deviations. If the spread becomes too high, it must be the case that income is low enough and default becomes a more attractive option. In the latter, however, a sudden fall in the belief may cause a downward shift of the entire bond price schedule, as shown in Figure 4. Spreads may then shoot up, while income remains relatively high, making default unattractive because of the nonlinear punishment function. As a result, the government sells bonds at steep discounts until one of the two outcomes occurs: either income falls enough to make default attractive or debt is reduced enough and spreads return to a desirable level.

The ability of the model to generate high values of the realized bond spread is akin to the result of Aguiar et al. (2022), who revisit the theory of rollover crises to rationalize the existence of "desperate deals." Crucially, here I show that a similar behavior can be obtained with fundamental factors. Online Appendix C provides a supplementary analysis of the drivers of equilibrium spreads. Specifically, it shows that movements in the belief are the main driver of spread volatility in the simulations. Moreover, this predominantly occurs through the belief's impact on the expected bond prices, rather than the next-period default probability.

E. *Event Analysis of the European Debt Crisis*

In this section I use the calibrated model to conduct an event study of the debt crisis in Portugal. I start with the benchmark AR(1) model and then move on to the predictions of the main model. I conclude by showing a counterfactual where agents have full information.

Benchmark Case: Standard AR(1).—I start by feeding the actual detrended GDP observations for Portugal into the benchmark off-the-shelf AR(1) version of the

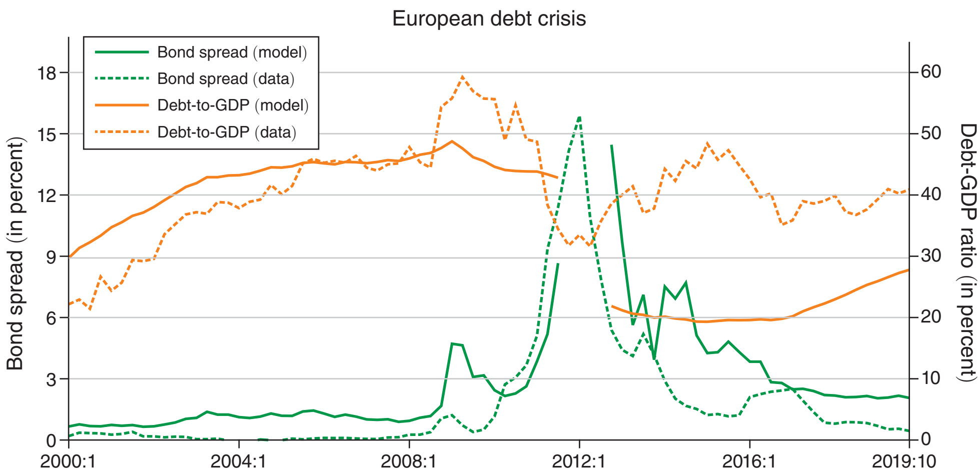

Figure 6. Event Analysis in the Model Based on a Standard AR(1) Process

Notes: To resolve the predicted default in this event study, I assume that the government reenters the market with an exogenous debt write-off. This aims to mimic the aid implied in the emergency loans that Portugal received from the European Commission and the IMF. Quantitatively, the write-off amounts to about 20 percent of the outstanding debt securities. Online Appendix D presents the details of this calculation.

model. Figure 6 presents the predicted evolution of debt-to-GDP ratio and the bond spread in that model and in the data (this is the same time series as in Figure 1). The economy is started in the first quarter of 1998 with the actual debt and GDP levels from the data. I use the model-implied decision rules and bond prices to generate endogenous responses to the realized path of income shocks.

Figure 6 highlights all of the problems with an off-the-shelf AR(1) model that have been highlighted in the preceding sections. The government accumulates debt too fast, at least until 2003, due to the fact that the model requires a low value of the discount factor in order to hit the targets of average debt and average spread. Throughout this time, the bond spread is strictly positive at around 1 percent and actively responds to current income shocks, including the recession of 2002–2003. When the Great Recession starts, the spread jumps up to 4.7 percent in 2009:I due to the extreme negative shock that hits Portuguese GDP. The government reduces its debt sharply, which causes the spread to fall back. These predictions are at odds with the data—the bond spread was virtually zero up until 2009, and debt accumulation was slower. In 2009 the 5-year bond spread rose to 1.2 percent, while the government actually increased its external debt-GDP ratio. Finally, the model predicts a further rise in the spread and a sequence of defaults that start in the last quarter of 2011. Notably, the model falls short of replicating the observed peak spread level of 16 percent (the highest it can get before default is 8.7 percent in 2011:III) and mispredicts the key variables in the post-2016 recovery period.

Model with Disaster Risk and Learning.—Now I conduct an event analysis for my main model. The upper panel of Figure 7 presents the predicted evolution of

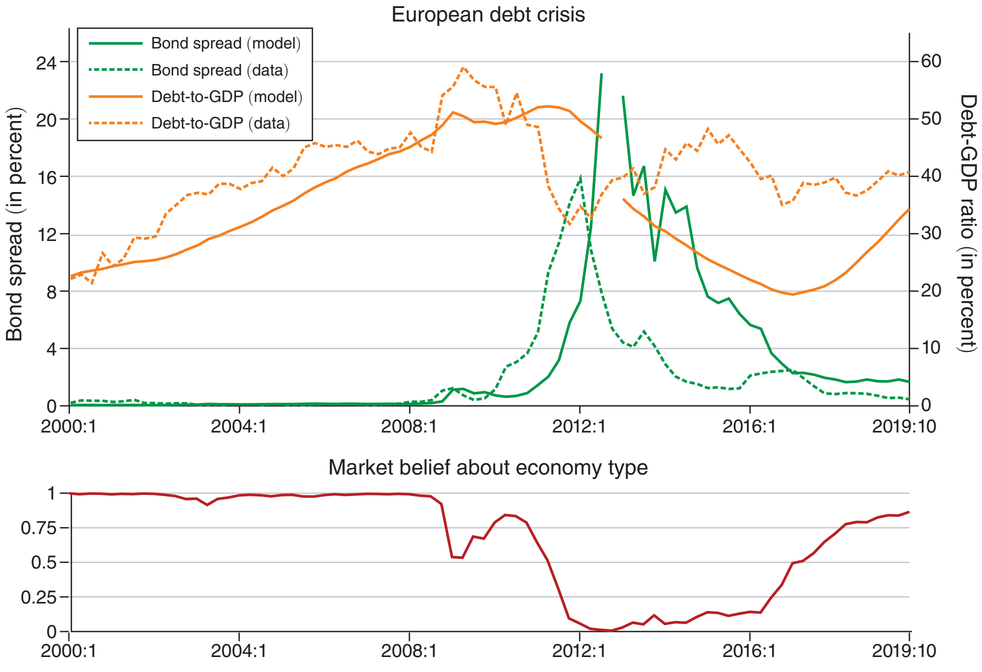

Figure 7. Event Analysis in the Model with Disaster Risk and Learning

Note: To resolve the default, I apply the same assumption as described in Figure 6 and online Appendix D.

debt-to-GDP ratio and bond spread for the actual path of Portuguese GDP realizations. The lower panel tracks the evolution of agents' belief about the economy type. The model offers an improvement on most fronts of the analysis relative to the benchmark in Figure 6. The pace of debt accumulation prior to 2008 is slower and consistent with the data due to the fact that we are able to use a much higher discount factor. At the same time, the predicted bond spread is essentially zero because investors have a near-certainty that the economy is operating in the high regime in which a default almost never happens. When the negative shock of 2009:I hits, the belief drops on impact but only partially, leading the spread to increase to 1.1 percent while debt is reduced in response, but then immediately picks up and stays on par with the data until the end of 2010. Finally, in 2011–2012 a new wave of low GDP shocks hits the economy, which causes the belief to plunge. This contributes to a sharp increase in the bond spread, which is predicted to reach 13 percent in 2012:II and 23 percent in 2012:III, followed by a default. While in reality the Portuguese spread did not go that high, its upward path was halted by the successful bailout provided by the IMF and the European Commission. By contrast, the Greek 10-year bond spread exceeded 27 percent in February 2012, which shows that such high values of the spread are feasible in equilibrium, and here I argue that they can be rationalized by a model based on fundamental factors.

In the aftermath of the default, the government is readmitted to the market and sharply reduces the debt further, due to the belief of being in the depression regime.[25] As the belief picks up again starting from 2016, the predicted bond spread and debt both converge toward their data counterparts for the same reasons as in the pre-2008 period. This shows that, outside the period of Portugal's participation in the bailout program, the theory can also fit the recovery that followed the crisis, especially after 2016.[26]

The contrast in the predictions captured by Figures 6 and 7 stems from the difference in the path of expectations about future income. Online Appendix C constructs a counterpart to Figure 2 using both variants of the model. Unlike an off-the-shelf AR(1) variant, the learning model generates the pattern of gradual forecast revisions that resembles the evidence in Section IB.

Naturally, the model still falls short of replicating the data in a few ways. First, the government in the model reduces its debt in response to the shocks in 2009, which is due to the elevated likelihood of an underlying regime switch. In reality most European countries actively *increased* their debt during that time, as Figure 1 shows. Paluszynski and Stefanidis (forthcoming) argue that such "borrowing into debt crises" behavior may be due to frictions in adjusting government expenditure. Second, the model predicts a sovereign default in 2012, which is not unreasonable, given that Portugal received a bailout from the European Commission and the IMF covering over 40 percent of its GDP. How to model a lender of last resort for sovereigns is a subject of active debate, and thus I leave this extension for future research.

F. *Learning Matters: A Counterfactual with Full Information*

Now I show that learning is crucial for the model to generate sensible predictions about the actual debt crisis in Europe. Figure 8 presents the event study with a variant of the model in which agents have complete information about the underlying regime switches (online Appendix E describes the calibration details for this variant). As in the previous case, debt increases gradually prior to the Great Recession, while spreads remain at zero. In 2008:III upon learning about the regime switch, spread increases to 3.3 percent while the government embarks on a drastic debt reduction path. Interestingly, even though the debt is much lower in the second stage of the European crisis relative to the predictions in Figure 6 and Figure 7, the spread still shoots up to 33 percent in 2012:IV without causing a default. This is because the agents are aware that the economy is on a downward trajectory, but there are no belief swings that would magnify the income shock and push the government into default.

[25] The lack of such debt reduction in the data may be due to Portugal's continual participation in the EU-IMF bailout program, a force that is absent from the model. Notice that as Portugal officially exits the program in mid-2014, it begins to reduce its debt securities. This lasts until mid-2016 when the government, in the model and in the data alike, starts accumulating debt again.

[26] A few crucial factors are missing in the model to explain years 2012–2016, as highlighted by many of the studies in the introduction, in particular the negotiations over EU bailouts and equilibrium multiplicity.

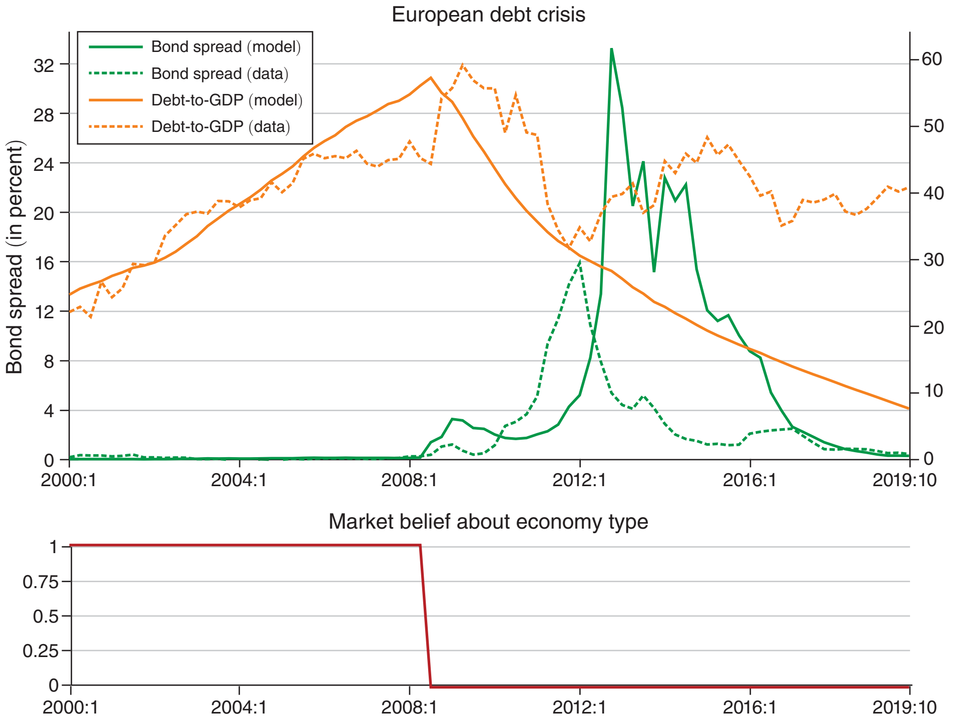

Figure 8. Event Analysis in the Model with Full Information about the Disaster Regime

IV. Conclusion

In their seminal contribution Lucas and Sargent (1979, 13) make the following remark about general equilibrium macroeconomic models:

> It has been only a matter of analytical convenience and not of necessity that equilibrium models have used the assumption of stochastically stationary shocks and the assumption that agents have already learned the probability distributions they face. Both of these assumptions can be abandoned, albeit at a cost in terms of the simplicity of the model.

This paper shows that learning about the probability distributions of future income shocks was an important driver of the European debt crisis. It impacted not only the movements in asset prices but also the real variables such as government debt. I show that an otherwise standard quantitative model of sovereign debt can be augmented to incorporate this learning process and match the evidence on the gradually evolving beliefs over time. As a result, we can obtain a delayed pattern of bond spread increases during the Great Recession in Europe.

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