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$m_e = 0.511 Mev$
$c = 3\times 10^8$ m/s
hc = 1240 eV nm

Problem 1

A 10 Mev electron strikes another electron (initially at rest).
(a) What is the relativistic momentum of each electron compared to the classical momentum.
Define : $\gamma = K/m_0c^2 = 10/0.511$
Before: $pc_{electron1} = \sqrt{\gamma(2+\gamma)}\,m_0c^2$; $pc_{electron2} = 0$ x component of relativistic momentum
Classical computation would use $p^2/2m = K,$ or $pc = \sqrt{2\gamma}\,m_0c^2$

(b) It it is observed that the first electron is scattered at an angle of 30 degrees, what is the Kinetic energy and scattering angle of the second electron. (Hint: use symmetry)
Conserve relativistic 4 momentum.
After: From symmetry y components of momentum are equal in magnitude, but opposite in sign. Hence scattering angles are 30 degrees above an below the x axis.
x components: $pc_{electron1}\cos(30) + pc_{electron2}\cos(30) = \sqrt{\gamma(2+\gamma)}\,m_0c^2$
y components: $pc_{electron1}\sin(30) - pc_{electron2}\sin(30) = 0$

Problem 2

A certain metal has a work function of 1.2 eV.
(a) What is the smallest frequency of electromagnetic radiation that will eject electrons.?

$E_{\max} = hf - \phi. \Rightarrow f = \phi/h$

(b) It the frequency of incident radiation is twice the minimum frequency, what will be the maximum speed of the ejected electrons.

$E_{\max} = hf - \phi = 2\phi - \phi = mv^2/2$ $v = \sqrt{2\phi/m} = (\sqrt{2\phi/mc^2}) \bullet c$ m/s

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Problem 3

(a) What are the velocities of an electron and a proton such that each will have the wavelength that corresponds to a 10 MeV photon.

$\lambda_{photon} = hc/hf = 1240/10^7 = 1.24 \times 10^{-4}$ nm.

$p_{electron} = h/\lambda_{photon} = \frac{m_o c(v/c)}{\sqrt{1-(v/c)^2}}$ where p is relativistic momentum,

$(h/\lambda_{photon})^2(1-(v/c)^2) = (m_o c)^2(v/c)^2$ or

$(v/c)^2 = (h/\lambda_{photon})^2/\{(h/\lambda_{photon})^2 + (m_o c)^2\} = (hf_{photon})^2/\{(hf_{photon})^2 + (m_o c^2)^2\}$

so

$(v/c)^2 = (\gamma)^2/\{(\gamma)^2 + 1\}$

where $\gamma = (hf_{photon})/(m_o c^2)$; $\gamma_{electron} = 10/0.0511$, $\gamma_{proton} = 10/938$

(b) Each of the objects above (photon, electron, proton – all of the same wavelength) are scattered by a pair of objects that produces a diffraction pattern. For the photon, the first minimum of the diffraction pattern is measured at an angle $\phi = 30$ degrees. What is the corresponding angle for the electron and the proton?

$\phi = 30$ degrees. (depends only on λ which is the same for all three objects

Problem 4

Consider an infinite square well potential with a width of 0.5 nanometers. For a particle of mass equal to that of the electron

(a) What is the "energy" of the "ground state above the base of the well. (Show how you determine this value)

Match boundary conditions such that wave function $sin(kr)$ is zero at boundaries. Hence $kL = n \bullet \pi$.

But $E = (\hbar^2 k^2)/2m = \hbar^2 n^2 \pi^2/2mL^2 = n^2\{\hbar^2(\pi)^2/2mL^2\} = n^2 E_0$

Ground state is when $n = 1$.

(b) What is the frequency of the radiation emitted if the electron "decays" from the third excited state to the ground state.

Third excited state is for $n = 4$.

Energy difference is $\Delta E = (4^2 - 1^2)E_0 = 15 \bullet E_0 = hf$. or $f = 15 \bullet E_0/h$

(c) sketch the wave functions for the first four admissable energy levels. What are the boundary conditions assumed.?

n=1: Ground state = half wavelength

n=2: state = full wavelength 1 interior node

n=3: state = 1.5 wavelengths, 2 interior nodes

n=4: state = 2 wavelengths, 3 interior nodes.

Match slope and value, (function $\Rightarrow$ 0 on the bondaries for an infinite well)