Krane Chapter 5

Problem 4

$$E_n = n^2 E_1$$
$$E_1 = 1.26 \text{ eV}$$
$$\Delta E_{31} = E_3 - E_1 = (n^2-1)E_1 = 8 \times 1.26 \text{ eV} = 10.1 \ eV$$
$$\Delta E_{41} = E_4 - E_1 = (n^2-1)E_1 = 15 \times 1.26 \text{ eV} = 18.9 \ eV$$

Problem 15

Divide the domain x into regions I, II, and III: $-\infty \Rightarrow 0, \ 0 \Rightarrow L, L \Rightarrow \infty$

The wave number $\sqrt{2m(E-U)/\hbar^2}$ is real in region II and imaginary in regions I and III

IN all three cases the possible solutions have the form

$$\boxed{\Psi(x) \ = Ae^{ikx} + Be^{-ikx}}$$

In region II the solutions are sin(kx) and cos(kx). In regions I and III the solutions are decaying or growing exponentials.

In region I the exponential with the minus sign in the exponent blows up as x goes to minus infinity, so its coefficent is set to zero. In region III the exponential with the plus sign in the exponent blows up as x goes to plus infinity, so its coefficent is set to zero.

From experience with the infinite well problem, the ground state wave function has no nodes (the wave function is even). The next excited state has 1 node. (Hence the wave function is odd). The next excited state has 2 nodes, and the wave function is even. The alternatiing process repeats itself with the wave functions for the nth excited state having n nodes.

Problem 19

$$E = E_0(n_x^2 + n_y^2 + n_z^2)$$
$$\Psi(1,1,1) = |1,1,1\rangle \Rightarrow E_0(1+1+1) = 3E_0$$

$$\Psi(2,1,1) = |2,1,1\rangle \Rightarrow E_0(4+1+1) = 6E_0$$
$$\Psi(1,2,1) = |1,2,1\rangle \Rightarrow E_0(1+4+1) = 6E_0$$
$$\Psi(1,1,2) = |1,1,2\rangle \Rightarrow E_0(1+1+4) = 6E_0$$

$$\Psi(2,2,1) = |2,2,1\rangle \Rightarrow E_0(4+4+1) = 9E_0$$
$$\Psi(1,2,2) = |1,2,2\rangle \Rightarrow E_0(1+4+4) = 9E_0$$
$$\Psi(2,1,2) = |2,1,2\rangle \Rightarrow E_0(4+1+4) = 9E_0$$

$$\Psi(3,1,1) = |3,1,1\rangle \Rightarrow E_0(9+1+1) = 11E_0$$
$$\Psi(1,3,1) = |1,3,1\rangle \Rightarrow E_0(1+9+1) = 11E_0$$
$$\Psi(1,1,3) = |1,1,3\rangle \Rightarrow E_0(1+1+9) = 11E_0$$

$$\Psi(2,2,2) = |2,2,2\rangle \Rightarrow E_0(4+4+4) = 12E_0$$

etc.

Problem 34

Divide the domain x into regions I, II, and III: $-\infty \Rightarrow 0, \ 0 \Rightarrow L, L \Rightarrow \infty$

The wave number $\sqrt{2m(E-U)/\hbar^2}$ is imaginary in region II and real in regions I and III

IN all three cases the possible solutions have the form

$$\boxed{\Psi(x) \;= Ae^{ikx} + Be^{-ikx}}$$

In region I or III the solutions are sin(kx) and cos(kx). In region II the solutions are decaying or growing exponentials.

region I:
$\Psi = Ae^{ikx} + Be^{-ikx}$ with $k = \sqrt{2mE/\hbar^2}$
region II:
$\Psi = Ce^{Kx} + De^{-Kx}$ with $K = \sqrt{2m(U-E)/\hbar^2}$
region III:
$\Psi = Fe^{ikx} + Ge^{-ikx}$ with $k = \sqrt{2mE/\hbar^2}$
but $G \Rightarrow 0$ under the hypothesis that there are no waves returning from infinity.

Match amplitudes and slopes at x= 0 and x = L

At x= 0:
Amplitude match: $A + B = C + D$
Slope match: $ik(A-B) = K(C-D)$

At x= L:
Amplitude match: $Ce^{KL} + De^{-KL} = Fe^{ikL}$
Slope match: $K\{Ce^{KL} - De^{-KL}\} = ikFe^{ikL}$

Solve the 4 simultaneous equations for the 4 unknowns B, C, D, F in terms of A

Problem 35

Divide the domain x into regions I, II, and III: $-\infty \Rightarrow 0,\ 0 \Rightarrow L, L \Rightarrow \infty$
The wave number $\sqrt{2m(E-U)/\hbar^2}$ is real in region II and real in regions I and III
The wavelength in region II is longer than the wavelength in region I and III.
IN all three cases the possible solutions have the form

$$\boxed{\Psi(x) \;= Ae^{ikx} + Be^{-ikx}}$$

In region I, II or III the solutions are sin(kx) and cos(kx).
region I:
$\Psi = Ae^{ikx} + Be^{-ikx}$ with $k = \sqrt{2mE/\hbar^2}$
region II:
$\Psi = Ce^{ik_2x} + De^{-ik_2x}$ with $k_2 = \sqrt{2m(E-U)/\hbar^2}$
region III:
$\Psi = Fe^{ikx} + Ge^{-ikx}$ with $k = \sqrt{2mE/\hbar^2}$
but $G \Rightarrow 0$ under the hypothesis that there are no waves returning from infinity.

Match amplitudes and slopes at x= 0 and x = L

At x= 0:
Amplitude match: $A + B = C + D$
Slope match: $ik(A-B) = ik_2(C-D)$

At x= L:
Amplitude match: $Ce^{ik_2L} + De^{-ik_2L} = Fe^{ikL}$
Slope match: $k_2\{Ce^{ik_2x} - De^{-ik_2x}\} = ikFe^{ikL}$

Solve the 4 simultaneous equations for the 4 unknowns B, C, D, F in terms of A