ECE 6364 Spring 2016 HW 09 Due 4/05

Problem 1.

In the derivation of the Wiener filter for restoring a continuous image under the continuous/continuous model, in the handout provided, show that

$$S_{fg}(u,v) = FT\{R_{fg}(x,y)\} = E\{f(\alpha,\beta)\, g(\alpha+x,\beta+y)\} = H^*(u,v)\, S_{ff}(u,v)$$

Problem 2.

In the derivation of the Wiener filter for restoring a continuous image under the continuous/continuous model in the handout provided, show that

$$S_{gg}(u,v) = FT\{R_{gg}(x,y)\} = FT\{E\{g(\alpha,\beta)\, g(\alpha+x,\beta+y)\}\} = S_{ff}(u,v)\,|H(u,v)|^2 + S_{nn}(u,v)$$

Problem 3

An observed image $g(x,y)$ is modeled as having been formed from a non-zero-mean image $u(x,y)$ that has been degraded by motion-blur and additive zero-mean white noise $n(x,y)$ with $E\{n(x,y)\, n(x-\gamma,y-\beta)\} = \mathrm{cov}_n(\gamma,\beta) = \alpha^2\, \delta(x-\gamma,y-\beta)$ according to

$$g(x,y) = \frac{1}{T}\int_0^T u(x-vt,y)\,dt + n(x,y)$$

If the mean $\bar{u}(x,y)$ of $u(x,y)$ is known, and the covariance $\mathrm{cov}_u(\gamma,\beta)$ of $u(x,y)$ is modeled as stationary

$$\mathrm{cov}_u(\gamma,\beta) = \sigma^2 \exp[-0.05(|x|+|y|)]$$

find the Fourier Transform $M(\zeta_1,\zeta_2)$ of the Wiener-filter shift-invariant impulse response $m(x,y)$ for restoring $u(x,y)$ from $g(x,y)$.

Problem 4.

1) Read in the cameraman TIF image into your matlab program and sub-sample it to size 128x128 as in previous HWs.
2) Form a mask by sampling a 2-D Gaussian function whose standard deviation is 0.9 pixel widths. What PxQ size mask should you use? Surely, choose odd numbers for P and Q so that you sample the Gaussian at (x,y)=(0.0,0.0). Normalize the mask to sum to 1.0;
3) Convolve the mask with the subsampled cameraman image $\vec{\mathbf{f}}$ to form a blurred image $\underline{\mathbf{H}}\vec{\mathbf{f}}$ the same size as the cameraman image. Make sure to normalize your mask for each output pixel so that the portion of the mask overlaying the image sums to 1.0 as in HW 08. Do you need to re-scale the blurred image 0-255? Should you?
4) Now add Gaussian white noise $\vec{\mathbf{n}}$ to the blurred image $\underline{\mathbf{H}}\vec{\mathbf{f}}$ via $\vec{\mathbf{g}} = \underline{\mathbf{H}}\vec{\mathbf{f}} + \vec{\mathbf{n}}$ to form three additional noisy blurred images $\vec{\mathbf{g}}$ that have SNRs of 100.0, 10.0, and 1.0. Use $\mathbf{SNR} = \dfrac{\sqrt{\frac{1}{MN}\|\vec{\mathbf{f}}\|^2}}{\sigma}$ where the image $\vec{\mathbf{f}}$ is of size MxN and σ is the standard deviation for the additive uncorrelated Gaussian noise $\vec{\mathbf{n}}$.
5) Matlab's Wiener filter function is deconvwnr(A, PSF, NSR), where A is the blurred image, PSF is the point-spread function, and NSR is the noise-power-to-signal-power ratio. Restore the three blurred noisy images using NSR = 0. When NSR=0, the Wiener restoration filter is equivalent to an ideal inverse filter, that is extremely sensitive to noise in the input image.
6) Now restore the three blurred noisy images using a Wiener filter with an estimate EstNSR of the noise-power-to-signal-power ratio.deconvwnr(blurred_noisy, PSF, EstNSR). How did you get your estimate EstNSR? Did you use the SNR values that you simulated the noisy blurred images with? If so, how would you ever have such knowledge in an actual application? How can you estimate the noise using a "flat" (constant) region of the noisy blurred image?
7) Write up a short report containing the restored images, attach it to your HW..