ECE 6364 Spring 2016 HW 08 due 3/29

Problem 1.

Fundamentals of Digital Image Processing - Jain: Problem 2.4 a,b,c,d

2.4 In each of the following systems find the impulse response and determine whether or not the system is linear, shift invariant, FIR, or IIR.

a. $y(m, n) = 3x(m, n) + 9$

b. $y(m, n) = m^2 n^2 x(m, n)$

c. $y(m, n) = \sum_{m'=-\infty}^{m} \sum_{n'=-\infty}^{n} x(m', n')$

d. $y(m, n) = x(m - m_0, n - n_0)$

e. $y(m, n) = \exp\{-|x(m, n)|\}$

f. $y(m, n) = \sum_{m'=-1}^{1} \sum_{n'=-1}^{1} x(m', n')$

g. $y(m, n) = \sum_{m'=0}^{M-1} \sum_{n'=0}^{N-1} x(m', n') \exp\left\{-j\frac{2\pi mm'}{M}\right\} \exp\left\{-j\frac{2\pi nn'}{N}\right\}$

Problem 2. Given 2-D discrete input image $x(m,n)$ and 2-D discrete output image $y(m,n)$ related by

$y(m,n) = x(m,0) - 3x(m,n)$

a) Show whether or not this is a linear system.
b) Find the impulse response of the system
c) Show whether or not the system is shift-invariant.
d) Show whether or not this system is causal.
e) Show (don't just guess) whether the system is an FIR or an IIR system.

Problem 3.

Given the 2-D image row-ordered $\vec{\mathbf{a}}$, $\underline{\mathbf{A}} = \begin{bmatrix} 120 & 189 & 112 \\ 200 & 126 & 181 \\ 110 & 194 & 124 \end{bmatrix}$ compute a sample estimate for a model of the mean of the image $E\{\vec{\mathbf{a}}\} = m_0 \vec{1}$, and compute a sample estimate for a model of the covariance $\underline{\mathbf{C}}$ of the image,

$$\underline{\mathbf{C}} = E\left\{ \left[\vec{\mathbf{a}} - m_0 \vec{1}\right] \left[\vec{\mathbf{a}} - m_0 \vec{1}\right]^t \right\}$$

$$\underline{\mathbf{C}} = \begin{bmatrix} \alpha & \beta & 0 & \beta & \gamma & 0 & 0 & 0 & 0 \\ \beta & \alpha & \beta & \gamma & \beta & \gamma & 0 & 0 & 0 \\ 0 & \beta & \alpha & 0 & \gamma & \beta & 0 & 0 & 0 \\ \beta & \gamma & 0 & \alpha & \beta & 0 & \beta & \gamma & 0 \\ \gamma & \beta & \gamma & \beta & \alpha & \beta & \gamma & \beta & \gamma \\ 0 & \gamma & \beta & 0 & \beta & \alpha & 0 & \gamma & \beta \\ 0 & 0 & 0 & \beta & \gamma & 0 & \alpha & \beta & 0 \\ 0 & 0 & 0 & \gamma & \beta & \gamma & \beta & \alpha & \beta \\ 0 & 0 & 0 & 0 & \gamma & \beta & 0 & \beta & \alpha \end{bmatrix}$$

where $\alpha = E\left\{\left[\underline{\mathbf{A}}_{i,j} - E\{\underline{\mathbf{A}}_{i,j}\}\right]^2\right\}$

$$\beta = E\left\{\left[\underline{\mathbf{A}}_{i,j} - E\{\underline{\mathbf{A}}_{i,j}\}\right]\left[\underline{\mathbf{A}}_{i+1,j} - E\{\underline{\mathbf{A}}_{i+1,j}\}\right]\right\} = E\left\{\left[\underline{\mathbf{A}}_{i,j} - E\{\underline{\mathbf{A}}_{i,j}\}\right]\left[\underline{\mathbf{A}}_{i,j+1} - E\{\underline{\mathbf{A}}_{i,j+1}\}\right]\right\}$$

$$\gamma = E\left\{\left[\underline{\mathbf{A}}_{i,j} - E\{\underline{\mathbf{A}}_{i,j}\}\right]\left[\underline{\mathbf{A}}_{i+1,j+1} - E\{\underline{\mathbf{A}}_{i+1,j+1}\}\right]\right\} = E\left\{\left[\underline{\mathbf{A}}_{i,j} - E\{\underline{\mathbf{A}}_{i,j}\}\right]\left[\underline{\mathbf{A}}_{i+1,j-1} - E\{\underline{\mathbf{A}}_{i+1,j-1}\}\right]\right\}$$

For example, the sample estimate of m_0 in $E\{\vec{\mathbf{a}}\} = m_0\vec{1}$ is $\hat{m}_0 = (1/9)(120+189+112+..+124)$ as you would expect.

The sample estimate for $\alpha = E\left\{\left[\underline{\mathbf{A}}_{i,j} - E\{\underline{\mathbf{A}}_{i,j}\}\right]^2\right\}$ is $\hat{\alpha} = (1/9)([120-\hat{m}_0]^2 + [189-\hat{m}_0]^2 + .. + [124-\hat{m}_0]^2)$ as you would expect.

The sample estimate for $\beta = E\left\{\left[\underline{\mathbf{A}}_{i,j} - E\{\underline{\mathbf{A}}_{i,j}\}\right]\left[\underline{\mathbf{A}}_{i+1,j} - E\{\underline{\mathbf{A}}_{i+1,j}\}\right]\right\} = E\left\{\left[\underline{\mathbf{A}}_{i,j} - E\{\underline{\mathbf{A}}_{i,j}\}\right]\left[\underline{\mathbf{A}}_{i,j+1} - E\{\underline{\mathbf{A}}_{i,j+1}\}\right]\right\}$ is the average of terms like $(\underline{\mathbf{A}}_{i,j} - \hat{m}_0)(\underline{\mathbf{A}}_{i+1,j} - \hat{m}_0)$ for which there are 6 terms and terms like $(\underline{\mathbf{A}}_{i,j} - \hat{m}_0)(\underline{\mathbf{A}}_{i,j+1} - \hat{m}_0)$ for which there are another 6 terms; yielding a total of 12 terms.

$\hat{\beta} = (1/12)[(120-\hat{m}_0)(189-\hat{m}_0) + (189-\hat{m}_0)(112-\hat{m}_0) + (200-\hat{m}_0)(126-\hat{m}_0)....]$.

Compute $\hat{m}_0$, $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ given image $\underline{\mathbf{A}} = \begin{bmatrix} 120 & 189 & 112 \\ 200 & 126 & 181 \\ 110 & 194 & 124 \end{bmatrix}$

<u>Problem 4.</u>

1) Read in the cameraman TIF image into your matlab program and sub-sample it to size 128x128 as in a previous HW.
2) Form a mask by sampling a 2-D Gaussian function whose standard deviation is 0.6 pixel widths. What PxQ size mask should you use? Surely, choose odd numbers for P and Q so that you sample the Gaussian at (x,y)=(0.0,0.0). Normalize the mask to sum to 1.0;
3) Convolve the mask with the subsampled cameraman image $\vec{\mathbf{f}}$ to form a blurred image $\underline{\mathbf{H}}\vec{\mathbf{f}}$ the same size as the cameraman image. Make sure to normalize your mask for each output pixel so that the portion of the mask overlaying the image sums to 1.0 as in a previous HW.
4) Now add Gaussian white noise $\vec{\mathbf{n}}$ to the blurred image $\underline{\mathbf{H}}\vec{\mathbf{f}}$ via $\vec{\mathbf{g}} = \underline{\mathbf{H}}\vec{\mathbf{f}} + \vec{\mathbf{n}}$ to form three additional noisy blurred images $\vec{\mathbf{g}}$ that have SNRs of 1000.0, 100.0, and 10.0. Use $\mathbf{SNR} = \dfrac{\sqrt{\frac{1}{MN}\|\vec{\mathbf{f}}\|_2^2}}{\sigma}$ where the image $\vec{\mathbf{f}}$ is of size MxN and σ is the standard deviation for the additive uncorrelated Gaussian noise $\vec{\mathbf{n}}$. Truncate pixels <0 to 0, and truncate pixels >255 to 255.
5) Now restore the blurred image having no noise in step (3) and restore the three noisy images. Use `P = inv(H); then Compute` $\hat{\mathbf{f}} = inv(\underline{\mathbf{H}})\,(\underline{\mathbf{H}}\vec{\mathbf{f}})$ for the blurred image with no noise, and compute $\hat{\mathbf{f}} = inv(\underline{\mathbf{H}})\,\vec{\mathbf{g}}$ for the three noisy-blurred images.
6) Now use the matlab pseudo-inverse $pinv(\underline{\mathbf{H}})$ `instead of the inverse` $inv(\underline{\mathbf{H}})$ to restore the four images. Compare the results and infer conclusions.