# <u>ECE 6364 Spr 2016 HW 5 due 2/23</u>

**<u>Problem 1.</u>**
Fundamentals of Digital Image Processing - Jain: Problem 4.3

Note the typo in book: the reconstruction filter has bandwidths of $\left(\frac{1}{2\Delta x}, \frac{1}{2\Delta y}\right) \quad Hz$

**<u>Problem 2.</u>**
Fundamentals of Digital Image Processing  Jain Problem 4.12

**<u>Problem 3.</u>** In the course handout on orthonormal expansions, the squared error $e^2_{M,N}$ between a function (continuous image) $f(x,y)$ and its approximation $\hat{f}(x,y)$ is given by

$$e^2_{M,N} = \int_\Omega\int \left| f(x,y) - \hat{f}(x,y)\right|^2 dx\,dy = \int_0^{x_0}\int_0^{y_0} \left| f(x,y) - \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} a_{m,n}\ \phi_{m,n}(x,y)\right|^2 dx\,dy$$

The proof that $e^2_{M,N}$ is minimized by choosing the coefficients as $a_{m,n} = \int_\Omega\int f(x,y)\,\phi^*_{m,n}(x,y)\,dx\,dy$ does so by considering the use of any other set of coefficients $\{b_{m,n}\}$ and showing that the coefficients $\{b_{m,n}\}$ will produce larger squared error; i.e.

$$\int_\Omega\int \left| f(x,y) - \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} b_{m,n}\ \phi_{m,n}(x,y)\right|^2 dx\,dy \ \geq\ \int_\Omega\int \left| f(x,y) - \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} a_{m,n}\ \phi_{m,n}(x,y)\right|^2 dx\,dy\,.$$

In that proof, the term $\beta = \int_\Omega\int [B][C]^* dx\,dy = \int_\Omega\int \left[ f - \sum_{p=0}^{M-1}\sum_{q=0}^{N-1}\left[a_{p,q} - b_{p,q}\right]\phi_{p,q}\right]\left[ f - \sum_{i=0}^{M-1}\sum_{j=0}^{N-1} a_{i,j}\ \phi_{i,j}\right]^* dx\,dy$ is used to factor

$\int_\Omega\int \left| f(x,y) - \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} b_{m,n}\ \phi_{m,n}(x,y)\right|^2 dx\,dy$ as $\int_\Omega\int \left| f(x,y) - \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} b_{m,n}\ \phi_{m,n}(x,y)\right|^2 dx\,dy = e^2_{M,N} + \gamma + \beta + \beta^*$

where $e^2_{M,N}$ is the squared error from using the $\{a_{m,n}\}$ coefficients.

Show that $\beta = \beta^* = 0$