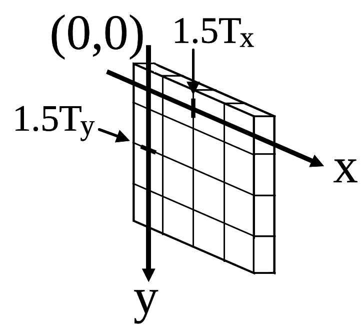
ECE 6364 Spring 2016 HW 04 due 2/16

Problem 1.

Recall, given a continuous image $g(x,y)$, an ideal point sample of $g(x,y)$ is simply $g(mT_x, nT_y)$ where T_x and T_y are the sample spacing in the x and y directions . Mathematically, we can form $g(mT_x, nT_y)$ as $g(mT_x, nT_y) = \int_{-\infty}^{+\infty}\int g(x,y)\ \delta(x - mT_x, y - nT_y)\ dx\ dy$, but this is simply the same as evaluating $g(x,y)$ at the point $(x,y) = (mT_x, nT_y)$.

Consider a CCD. Let us set up our coordinate-system so that (x,y)=(0,0) is at the very *center* of the upper-left cell of the CCD. Then the CCD forms a discrete image by integrating the photons from some continuous image $f(x,y)$ that is projected onto the CCD by the lens. The $(m,n)^{th}$ sample $f_d(m,n)$ from the CCD in formed by integrating photons from within the continuous region of $f(x,y)$ that corresponds to the $(m,n)^{th}$ cell of the CCD. We can express this with the equation



$$f_d(m,n) = \int_{(n-0.5)T_y}^{(n+0.5)T_y} \int_{(m-0.5)T_x}^{(m+0.5)T_x} f(\gamma,\beta)\quad d\gamma\ d\beta$$

Let us define a window function $w_{T_x,T_y}(x,y) = \begin{cases} 1 & |x| \le 0.5T_x \quad and \quad |y| \le 0.5T_y \\ 0 & else \end{cases}$

(a) Draw a 2-D graph of $w_{T_x,T_y}(x,y)$, labeling pertinent points on the x and y axes.

Now, consider a second continuous function $g(x,y)$, related to $f(x,y)$ as

$$g(x,y) = \int_{-\infty}^{+\infty}\int f(\gamma,\beta)\ w_{T_x,T_y}(\gamma - x, \beta - y)\ d\gamma\ d\beta.$$

Notice:

(1) the integrand $f(\gamma,\beta)\ w_{T_x,T_y}(\gamma - x, \beta - y)$ is a product of two functions, each of which is a function of independent variables (γ,β), and the integral forms $g(x,y)$ by integrating the product over all values of (γ,β).

(2) $w_{T_x,T_y}(\gamma - x, \beta - y)$ is a function of (γ,β) formed by shifting $w_{T_x,T_y}(\gamma,\beta)$ by an amount (x,y) in the (γ,β) directions.

(3) $f_d(m,n) = g(mT_x, nT_y)$; i.e. the CCD-samples are equal to point-samples of the continuous image $g(x,y)$.

(4) The window function $w_{T_x,T_y}(x,y)$ is symmetric about the origin, so that $w_{T_x,T_y}(\gamma - x, \beta - y) = w_{T_x,T_y}(x - \gamma, y - \beta)$ by which

$$g(x,y) = \int_{-\infty}^{+\infty}\int f(\gamma,\beta)\ w_{T_x,T_y}(x - \gamma, y - \beta)\ d\gamma\ d\beta = f(x,y) ** w_{T_x,T_y}(x,y)\ .$$

By taking the continuous Fourier transform of both sides of the above equation, we obtain

$G(u,v) = F(u,v)\, W_{T_x,T_y}(u,v)$

(b) To show that $g(x,y)$ is a low-pass version of $f(x,y)$, compute the Fourier transform $W_{T_x,T_y}(u,v)$ of $w_{T_x,T_y}(x,y)$. Graph $\left|W_{T_x,T_y}(u,v)\right|$. Label pertinent points on the u,v axes of the graph. Find the particular values of u,v where $\left|W_{T_x,T_y}(u,v)\right| = 0$. Compute and label the amplitude $\left|W_{T_x,T_y}(0,0)\right|$ as a function of T_x, T_y.

(c) Find the -3 dB frequencies u_{-3dB} and v_{-3dB} for $W_{T_x,T_y}(u,v)$ as a function of T_x and T_y; i.e. where
$\frac{1}{\sqrt{2}}\left|W_{T_x,T_y}(0,0)\right| = \left|W_{T_x,T_y}(u_{-3dB},0)\right| = \left|W_{T_x,T_y}(0,v_{-3dB})\right|$

Conclusions: You should now be able to see that
(1) The CCD-samples $f_d(m,n)$ are equivalent to ideal point-samples of a continuous image $g(x,y)$ where $g(x,y)$ is the low-pass-filtered version $f(x,y) ** w_{T_x,T_y}(x,y)$ of the continuous image $f(x,y)$ that is projected onto the CCD by the lens.
(2) The -3 dB cutoff frequencies of the lowpass filter $w_{T_x,T_y}(x,y)$ are important from the viewpoint of the sampling theorem. These frequencies, as you computed in 4c above, are sufficiently low as to preclude aliasing when a CCD forms a discrete image $\{f_d(m,n)\}$ whose samples are equivalent to point samples of $f(x,y) ** w_{T_x,T_y}(x,y)$, a lowpass filtered $f(x,y)$.
(3) If you consider $W_{T_x,T_y}(u,v)$ as approximating an ideal lowpass filter whose cutoff frequency is equal to the -3 dB cutoff freq of $W_{T_x,T_y}(u,v)$, then the CCD sample spacing is sufficiently small as to recover the lowpass image $g(x,y)$ from the point samples $\{f_d(m,n)\}$. That is, you cannot recover $f(x,y)$ from $\{f_d(m,n)\}$, but you can recover $g(x,y)$ where $g(x,y) = f(x,y) ** w_{T_x,T_y}(x,y)$ from $\{f_d(m,n)\}$.
(4) A CCD has a sort of self-regulating lowpass filter $W_{T_x,T_y}(u,v)$ that is proportional to the size of the CCD cells. If the cells are larger, then the -3 dB cutoff frequencies of $W_{T_x,T_y}(u,v)$ are lower, making the image $g(x,y)$ that can be recovered from $\{f_d(m,n)\}$ a more lowpass filtered version of $f(x,y)$ than would be the case if the cells of the CCD were smaller.

Problem 2.

Let $\underline{\mathbf{A}} = \begin{matrix} [0] & 1 & 2 & 3 \\ 4 & 4 & 4 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 9 & 9 & 9 \end{matrix}$ and $\underline{\mathbf{M}} = \begin{matrix} a & b & c \\ d & [e] & f \\ g & h & i \end{matrix}$ and $\underline{\mathbf{A}} ** \underline{\mathbf{M}} = \underline{\mathbf{C}} = \begin{matrix} [x] & x & x & x \\ x & x & x & x \\ x & x & x & x \\ x & x & x & x \end{matrix}$ where C is the central part of the linear convolution such that the size of the output image C is the same size as input image A.
Column order 4x4 image $\underline{\mathbf{A}}$ into 16x1 vector $\vec{\mathbf{a}}$ and column-order 4x4 result $\underline{\mathbf{C}}$ into vector $\vec{\mathbf{c}}$ to form the linear system $\vec{\mathbf{c}} = \underline{\mathbf{H}}\,\vec{\mathbf{b}}$. Fill-in the top 8 rows of $\underline{\mathbf{H}}$

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Problem 3.

Fundamentals of Digital Image Processing - Jain: Problem 4.2

Problem 4.

Part A: Add Gaussian white noise to your blurred image generated in HW03.
J = imnoise(I,'gaussian',M,V)
Use mean M=0.0 and variance V computed to yield the SNRs below.
Generate 4 images: one each at the four SNRs 100, 10, 1, 0.1.

Eq (1) $$SNR = \frac{average\ \ image\ pixel}{\sqrt{\sigma_{noise}^{2}}}.$$

Display the four images. Comment on their appearance.

Part B: Form J-I and compute the sample average of the standard deviation of the noise in J.

Eq (2) $$\hat{\sigma}_{noise} = \sqrt{\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left[J_{m,n} - I_{m,n}\right]^{2}}$$

Now compute your sample average estimate of the SNR of the noisy blurred image using Eq (2).

Eq (3) $$estimated\ SNR = \frac{average\ \ image\ pixel}{\hat{\sigma}_{noise}}$$

In each case (SNRs 100, 10, 1, 0.1.) are the estimated SNR and the SNR you used to form
J = imnoise(I,'gaussian',M,V) close to the same value?
List the values you obtained.

Part C: Find the max and min pixel values in noisy image J.
Are any pixels less than zero?
Since the blurred noise-free image I has pixel values 0-255, does noisy blurred image J have any pixels greater than 255?
Write down your conclusions.