CHAPTER 21 . OPTIONS

Contents

I. INTRODUCTION

• BASIC TERMS

1. Option : <u>right</u> to buy (or sell) an asset at a fixed price on or before a given date
   Right → buyer of option has no obligation, seller of option is obligated
   Call → right to buy
   Put → right to sell
   Note: Option may be written on any type of asset => most common is stock

2. Exercising the option - buying or selling asset by using option

3. Strike (or exercise) price - price at which asset may be bought or sold

4. Expiration date - last date on which option may be exercised

5. European option - may be exercised only at expiration date

6. American option - may be exercised on or before expiration date

7. In-the-money - positive cash flow if exercised → call [put] =?

8. Out-of-the-money - negative cash flow if exercised → call [put] = ?

9. At-the-money - zero cash flow if exercised $\rightarrow$ call [put] = ?

II. VALUATION OF OPTIONS

A. Minimum Values of Options

1. Minimum Value of Call

A call option is an instrument with limited liability. If the call holder sees that it is advantageous to exercise it, the call will be exercised. If exercising it will decrease the call holder's wealth, the holder will not exercise it. Thus, the option cannot have negative value, because the holder cannot be forced to exercise it.

Therefore, $C \geq 0$

For an American call, the statement that $C \geq 0$ is dominated by a much stronger statement : $C_a \geq$ Max $(0, S-E)$. The expression Max $(0, S-E)$ means "Take the maximum value of the two arguments, zero or S-E"

To prove this, consider the following example.

Example]   Strike price   = $35
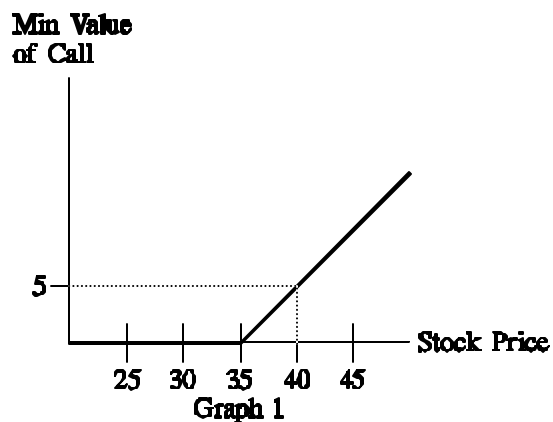   Stock price   = $39

• If the call were priced less than $4 -- say $3.

You could buy the call for $3, exercise it -- which would entail buying the stock for $35 -- and   then sell the stock for $39. This arbitrage transaction would give an immediate riskless profit of $1.

    All investors will would do this, which drives up the option price.

    When the price of the call reached $4, the transaction would no longer be profitable.

    THUS, $4 is the minimum price of the call.

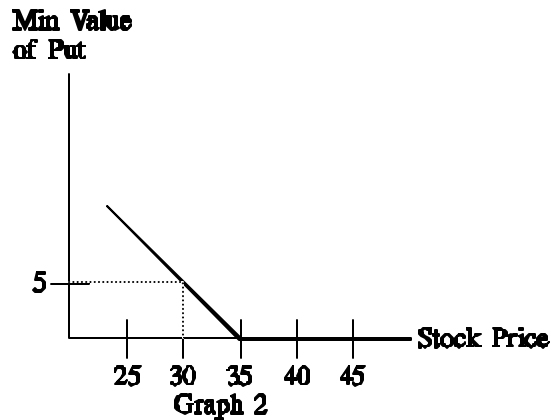Graph 1

If stock price = $40 => min. value of call =
If stock price = $30 => min. value of call =

2. Minimum Value of Put.
   Similar logic applies to put options.
   $P \geq 0$
   For an American put, the statement that $P \geq 0$ is dominated by a much stronger statement : $P_a \geq$ Max (0, E-S). The expression Max (0, E-S) means "Take the maximum value of the two arguments, zero or E-S"

Min Value
of Put

Graph 2

If stock value = $40 => min. value =
If stock value = $30 => min. value =

B. Maximum Values of Options

1. Maximum Value of Call
 $C_a \geq$ Max (0, S-E)
   From the equation above, even if the exercise price were zero, no one would pay more for the call than for the stock. Therefore, the maximum value of a call is the stock price.

2. Maximum Value of Call
   $P_a \geq$ Max (0, E-S)
   From the equation above, even if the exercise price were zero, no one would pay more for the put than for the stock. Therefore, the maximum value of a put is the stock price.

Graph 3
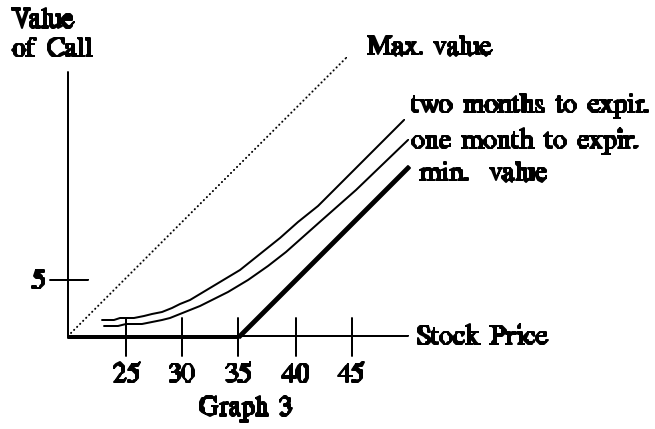
C. Determinants of Option Value

There are six factors affecting the price of a stock options.
1. The current stock price
2. The exercise [or strike] price
3. The time to expiration
4. The volatility of the stock price
5. The risk free interest rate
6. The dividends expected during the life of the option

: In this section, we consider what happens to option prices when one of tehse factors changes with all the others remaining fixed.

1. *Current stock price*
   $\rightarrow$ If it is exercised at some time in the future, the payoff from a call option will be the amount by which stock price exceeds the strike price. Therefore call option becomes more valuable as the stock price increases.
2. *Exercise price*
   $\rightarrow$ If it is exercised at some time in the future, the payoff from a call option will be the amount by which stock price exceeds the strike price. Therefore call option becomes less valuable the strike price increases.
3. *Time to expiration*
   $\rightarrow$ Both put and call American options become more valuable as the time to expiration increases. To see this, consider two options that differ only as far as the expiration date is concerned. The owner of the long-life option has all the exercise opportunities open to the owner of the short-life option -- and more. The long-life option must therefore always be worth at least as much as the short-life option.
   European put and call options do not necessarily become more valuable as the time to expiration increases. [why?]
4. *Volatility [or Variance] in stock prices*
   $\rightarrow$ Roughly speaking, the volatility of a stock price is a measure of how uncertain we are about future stock price movements.
   As volatility increases, the chance that the stock will do very well or very poorly increases. The owner of a call benefits from price increases but has limited downside risk in the event of price decreases. [why?] Therefore value of calls increases as volatility increases.
   The same logic applies to put options.
5. *Risk free interest*
   $\rightarrow$ If the stock price is expected to increase, an investor can choose to either buy the stock or buy the call. Purchasing the call will cost far less than purchasing the stock. The difference can be invested in risk-free bonds. If interest increase, the combination of calls and risk-free bonds will be more attractive. This means that the call price will tend to increase with increases in interest rates. However, when you sell the stock by exercising the put, you receive E dollars. If interest rates increase, the E dollars will have a lower present value. Thus, higher interest rates make put less attractive.

• Summary of the Effect on the Option Price of Increasing one variable holding other variables constant

	E.Call	E.Put	A.Call	A.Put
Stock price	+	-	+	-
Exercise price	-	+	-	+
Time to expiration	?	?	+	+
Volatility [Variance]	+	+	+	+
Risk-Free Rate	+	-	+	-

D. Black-Scholes Option Pricing Model (1973, BSOPM)

1. Assumptions

(1) No dividends (or other distributions)
(2) No transaction costs or taxes
(3) There is a constant risk free rate of interest at which people can borrow and lend any amount of money
(4) Unrestricted short selling of stocks is possible
(5) All option are European
(6) Security trading is continuous
(7) Stock price movements are random & continuous (no jumps)
(8) Stock returns are normally distributed

Note: Few of these assumptions are reasonable, but it works.

2. The model

$$C_0 = S_0[N(d_1)] - E * e^{-r_f t} [N(d_2)]$$

$$d_1 = \frac{\ln\left(\frac{S_0}{E}\right) + \left(r_f + \frac{s^2}{2}\right)t}{\sqrt{s^2 \times t}}$$

$$d_2 = d_1 - \sqrt{s^2 \times t}$$

C_0 = value of call today
S_0 = current stock price
E = exercise price of call
e = exponential function ≈ 2.7183
r_f = continuously compounded annual risk free rate (decimal)
   Note: APR
σ^2 = variance of continuously compounded annual return on stock (decimal)
t = time to expiration on annual basis
$N(d_1)$; $N(d_2)$ = value of the cumulative normal distribution of d_1 & d_2 respectively

Note: All variables are observable (in WSJ) except for σ^2
   r_f =>

   σ^2 =>

3. Using the BSOPM

(1) Some practice using N( )

$\rightarrow$ see Normal Distribution Tables

Ex. $d_1 = 0 \rightarrow$ at mean

$\Rightarrow N(d_1) = \textbf{?}$

(2) Putting it all together

Ex. You are considering purchasing a call option on Gurgle Baby Food Inc. This option has a strike price of $55 and expires 65 days from today. The return on a 65-day T-bill is 3.5% per year compounded continuously. Gurgle's current stock price is $58 per share and the standard deviation of returns on Gurgle is 46%. What is the value of this call option?
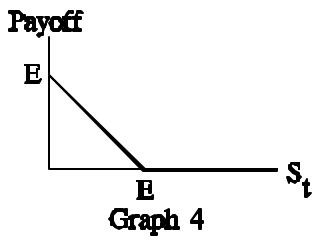
$s^2 =$

$t = 65/365$

$d_1 =$

$N(d_1) =$
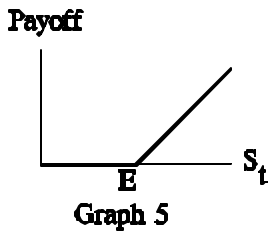
$N(d_2) =$

$\rightarrow C_0 =$

E. Put-Call Parity

1. Payoff for purchasing a put

    Payoff

    E

    E

    S_t

    Graph 4

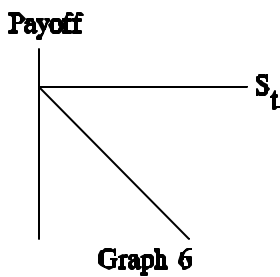2. Artificial Put

    → Buying a call, short selling stock, and lending the present value of the exercise price provides same payoff as buying a put.
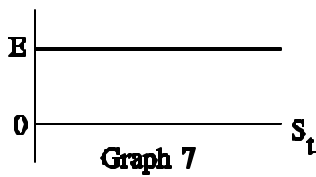
    (1) Buying call
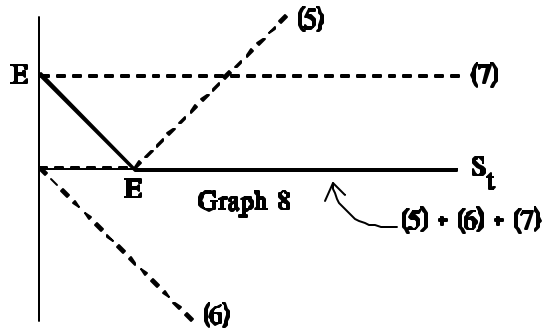
    Payoff

    E

    S_t

    Graph 5

    (2) Short selling stock

    Payoff

    S_t

    Graph 6

    (3) Lending the present value of the exercise price

    E

    0

    S_t

    Graph 7

(4) Buy call, short sell stock, and lend the present value of the exercise price

Graph 8

3. Using Put-Call Parity to value puts

→ Payoff from buying put =
> Payoff from buying call
> + Payoff from short selling stock,
> + Payoff from lending the present value of the exercise price

$$\rightarrow P_0 = C_0 - S_0 + E * e^{-r_f \times t}$$

Example) Option has a strike price of $55 and expires 65 days from today. The return on a 65-day T-bill is 3.5% per year compounded continuously. Gurgle's current stock price is $58 per share and the standard deviation of returns on Gurgle is 46%. What is the value of this put option?

Note: The call on Gurgle Baby Food is worth $6.14 (figured earlier)
→ P_0 = ?

**III. EXERCISE FOR THE PAYOFF DIAGRAMS.**

A. CALL OPTION

A **call option** is a contract giving its owner the right *[Not the obligation]* to buy a fixed amount of a specified underlying asset at a fixed price at any time or on or before a fixed date. For example, for an equity option, the underlying asset is the common stock. The fixed price is called the exercise price or the strike price. The fixed date is called the expiration date. On the expiration date, the value of a call on a per share basis will be the larger of **the stock price minus the exercise price or zero**.

a.. Long Call Option
One can think of the buyer of the option paying a premium (price) for the option to **buy** a specified quantity at a specified price any time prior to the maturity of the option. Consider an example. Suppose you buy an option to buy 1 UH stock at a price of $76. The option can be exercised on Dec. 19th.   The cost of the call is assumed to be $1. Let's tabulate the payoffs at expiration.

LONG CALL OPTION PAYOFF	
UH stock price on Dec. 19	Payoff on Option
71	-1
72	-1
73	-1
74	-1
75	-1
76	-1
77	0
78	1
79	2
80	3

Consider the payoffs diagrammatically. Notice that the payoffs are one to one after the price of the underlying security rises above the exercise price. When the security price is less than the exercice price, the call option is referred to as out of the money.
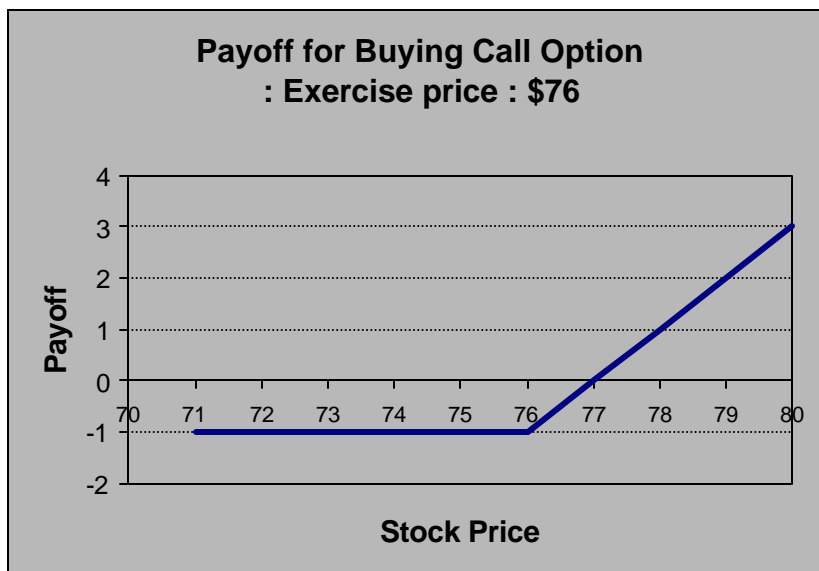
b. Short Call Option

Writing or "shorting" options have the exact opposite payoffs as purchased options.
The payoff table for the short call option is:

SHORT CALL OPTION PAYOFF	
UH stock price on Dec. 19	Payoff on Option
71	1
72	1
73	1
74	1
75	1
76	1
77	0
78	-1
79	-2
80	-3

Notice that the liability is potentially unlimited when you are writing call options.

Payoff for Selling Call Option : Exercise price : $76
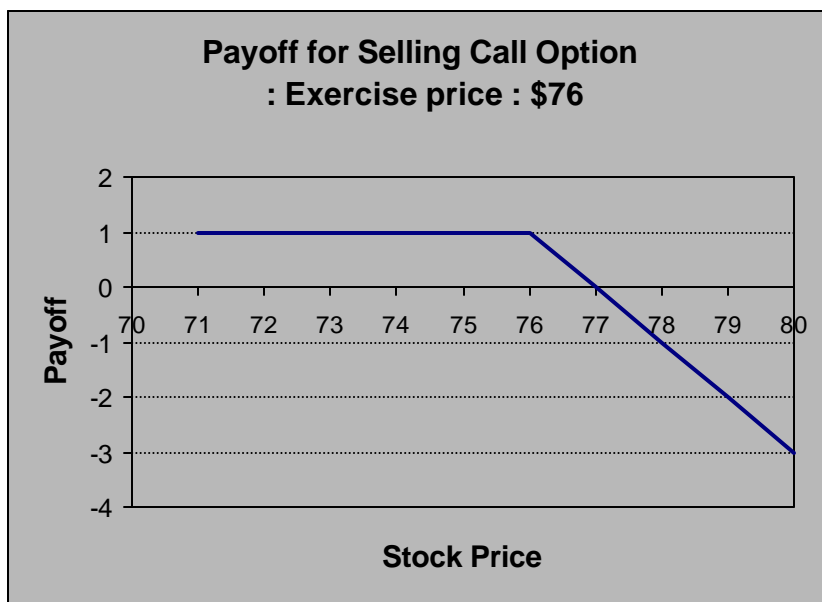
B. PUT OPTION

A **put option** is a contact giving its owner the right to sell a fixed amount of a specified underlying asset at a fixed price at any time on or before a fixed date. On the expiration date, the value of the put on a per share basis will be the larger of **the exercise price minus the stock price or zero**.

A. Long Put Option

One can think of the buyer of the put option as paying a premium (price) for the option to **sell** a specified quantity at a specified price any time prior to the maturity of the option. Consider an example of a put on the same UH stock. The exercise price is $76. You can exercise the option on Dec. 19.
Suppose that the cost of the put is $1.00.

LONG PUT OPTION PAYOFF	
UH stock price on Dec. 19	Payoff on Option
71	4
72	3
73	2
74	1
75	0
76	-1
77	-1
78	-1
79	-1
80	-1

The payoff from a long put can be illustrated. Notice that the payoffs are one to one when the price of the security is less than the exercise price.


Payoff for Buying Put Option : Exercise price : $76
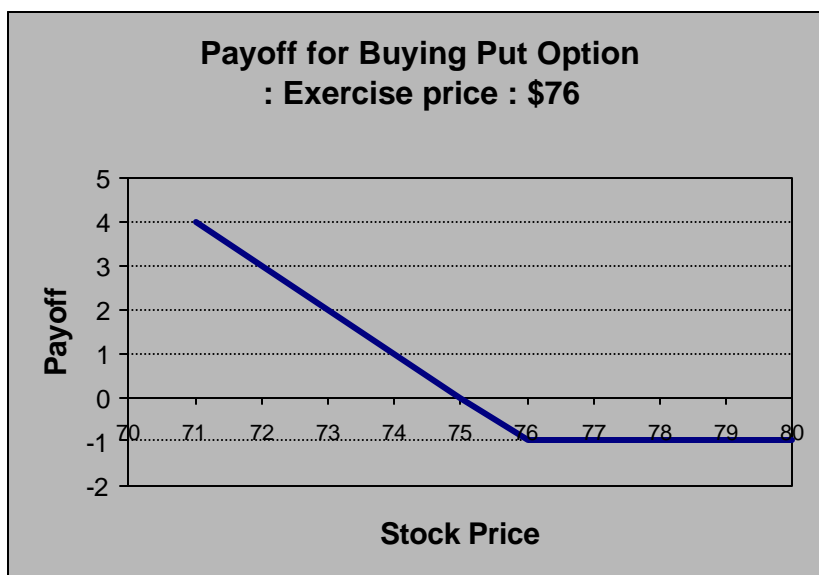
B. Short Put Option

Writing or "shorting" options have the exact opposite payoffs as purchased options.
The payoff table for the short put option is:

SHORT PUT OPTION PAYOFF	
UH stock price on Dec. 19	Payoff on Option
71	-4
72	-3
73	-2
74	-1
75	0
76	1
77	1
78	1
79	1
80	1

The payoff from a short put can be illustrated.

Payoff for Selling Put Option
: Exercise price : $76
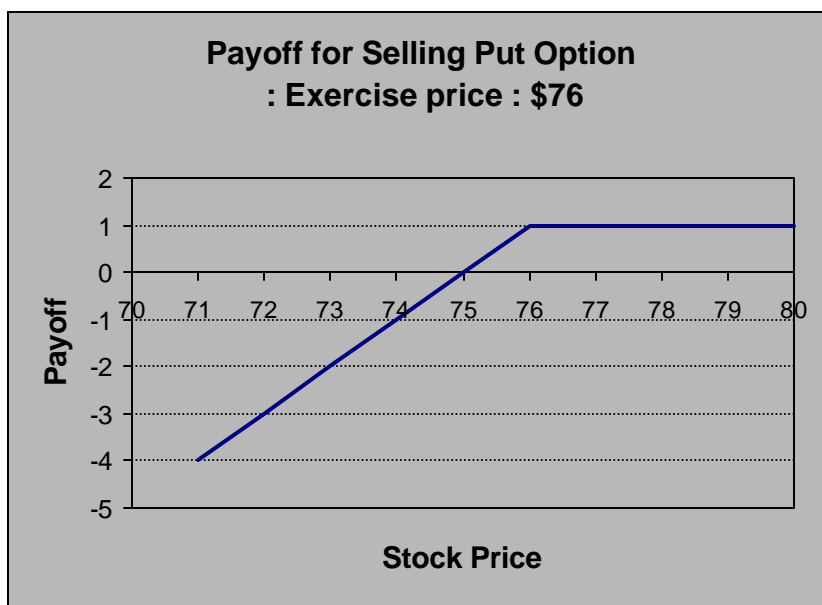
IV.   ASSIGNED PROBLEMS FROM THE TEXTBOOK
Q1 – Q13

V. SELF-EXERCISE PROBLEMS

Show the payoff diagram and table under the following conditions.
[Keep in mind that **Short position = Selling**   and   **Long position = Buying**]

For Question 1) and Question 2) only
• The call and put price are $1.
• The exercise prices for call and put option are $76
• You bought / or sold short a stock at $76.

Question 1)
Long position in a stock + Short position in a call
Question 2)
Short position in a stock + Short position in a put

Question 3)
Long position in a call with a exercise price of $73 and call price of $3
Short position in a call with a exercise price of $76 and call price of $1

Question 4)
Long position in a call and Long position in a put.
  -   Each with exercise price of $76 and option price of $1.

Question 5)
Short position in a call and Short position in a put.
  -   Each with exercise price of $76 and option price of $1.

Question 6)
  -   Long position in a call with exercise price of $71 and option price of $10.
  -   Long position in a call with exercise price of $81 and option price of $5
  -   Short position in **two** calls with exercise price of $76 and option price of $7.

Question 7)
  -   Long position in a put with a exercise price of $70 and option price of $2
  -   Long position in a call with a exercise price of $76 and option price of $1

Question 8)
  -   Short position in a put with a exercise price of $70 and option price of $2
  -   Short position in a call with a exercise price of $76 and option price of $1