

## CHAPTER 5 CHOICE UNDER UNCERTAINTY

### EXERCISES

**1. Consider a lottery with three possible outcomes: \$100 will be received with probability .1, \$50 with probability .2, and \$10 with probability .7.**

**a. What is the expected value of the lottery?**

The expected value,  $EV$ , of the lottery is equal to the sum of the returns weighted by their probabilities:

$$EV = (0.1)(\$100) + (0.2)(\$50) + (0.7)(\$10) = \$27.$$

**b. What is the variance of the outcomes of the lottery?**

The variance,  $\sigma^2$ , is the sum of the squared deviations from the mean, \$27, weighted by their probabilities:

$$\sigma^2 = (0.1)(100 - 27)^2 + (0.2)(50 - 27)^2 + (0.7)(10 - 27)^2 = \$841.$$

**c. What would a risk-neutral person pay to play the lottery?**

A risk-neutral person would pay the expected value of the lottery: \$27.

**3. Richard is deciding whether to buy a state lottery ticket. Each ticket costs \$1, and the probability of the following winning payoffs is given as follows:**

| Probability | Return |
|-------------|--------|
| 0.50        | \$0.00 |
| 0.25        | \$1.00 |
| 0.20        | \$2.00 |
| 0.05        | \$7.50 |

**a. What is the expected value of Richard's payoff if he buys a lottery ticket? What is the variance?**

The expected value of the lottery is equal to the sum of the returns weighted by their probabilities:

$$EV = (0.5)(0) + (0.25)(\$1.00) + (0.2)(\$2.00) + (0.05)(\$7.50) = \$1.025$$

The variance is the sum of the squared deviation from the mean, \$1.025, weighted by their probabilities:

$$\sigma^2 = (0.5)(0 - 1.025)^2 + (0.25)(1 - 1.025)^2 + (0.2)(2 - 1.025)^2 + (0.05)(7.5 - 1.025)^2, \text{ or}$$

$$\sigma^2 = \$2.812.$$

**b. Richard's nickname is "No-risk Rick." He is an extremely risk-averse individual. Would he buy the ticket?**

An extremely risk-averse individual will probably not buy the ticket, even though the expected outcome is higher than the price,  $\$1.025 > \$1.00$ . The difference in the expected return is not enough to compensate Rick for the risk. For example, if his wealth is \$10 and he buys a \$1.00 ticket, he would have \$9.00, \$10.00, \$11.00, and \$16.50, respectively, under the four possible outcomes. Let us assume that his utility function is  $U = W^{0.5}$ , where  $W$  is his wealth. Then his expected utility is:

$$EU = (0.5)(9^{0.5}) + (0.25)(10^{0.5}) + (0.2)(11^{0.5}) + (0.05)(16.5^{0.5}) = 3.157.$$

This is less than 3.162, which is the utility associated with not buying the ticket ( $U(10) = 10^{0.5} = 3.162$ ). He would prefer the sure thing, i.e., \$10.

- c. **Suppose Richard was offered insurance against losing any money. If he buys 1,000 lottery tickets, how much would he be willing to pay to insure his gamble?**

If Richard buys 1,000 tickets, it is likely that he will have \$1,025 minus the \$1,000 he paid, or \$25. He would not buy any insurance, as the expected return, \$1,025, is greater than the cost, \$1,000. He has insured himself by buying a large number of tickets.

- d. **In the long run, given the price of the lottery ticket and the probability/return table, what do you think the state would do about the lottery?**

In the long run, the state lottery will be bankrupt! Given the price of the ticket and the probabilities, the lottery is a money loser. The state must either raise the price of a ticket or lower the probability of positive payoffs.

6. **Suppose that Natasha's utility function is given by  $u(I) = I^{0.5}$ , where  $I$  represents annual income in thousands of dollars.**

- a. **Is Natasha risk loving, risk neutral, or risk averse? Explain.**

Natasha is risk averse. To show this, assume that she has \$10,000 and is offered a gamble of a \$1,000 gain with 50 percent probability and a \$1,000 loss with 50 percent probability. Her utility of \$10,000 is 3.162, ( $u(I) = 10^{0.5} = 3.162$ ). Her expected utility is:

$$EU = (0.5)(9^{0.5}) + (0.5)(11^{0.5}) = 3.158 < 3.162.$$

She would avoid the gamble. If she were risk neutral, she would be indifferent between the \$10,000 and the gamble; whereas, if she were risk loving, she would prefer the gamble.

You can also see that she is risk averse by plotting the function for a few values (see Figure 5.6) and noting that it displays a diminishing marginal utility. (Or, note that the second derivative is negative, again implying diminishing marginal utility.)

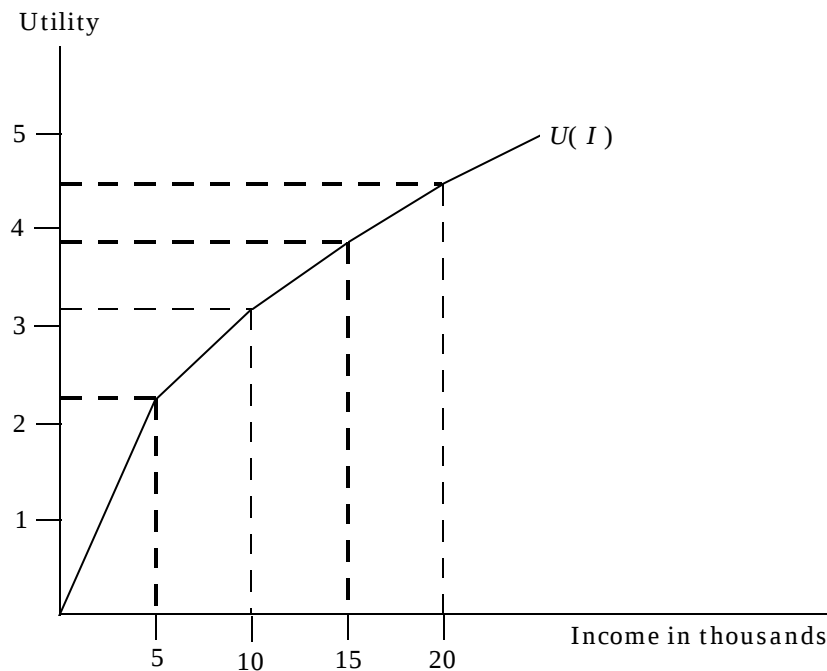


Figure 5.6

- b. **Suppose that Natasha is currently earning an income of \$10,000 ( $I = 10$ ) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a .5 probability of earning \$16,000, and a .5 probability of earning \$5,000. Should she take the new job?**

The utility of her current salary is  $10^{0.5}$ , which is 3.162. The expected utility of the new job is

$$EU = (0.5)(5^{0.5}) + (0.5)(16^{0.5}) = 3.118,$$

which is less than 3.162. Therefore, she should not take the job.

- c. **In (b), would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? (Hint: What is the risk premium?)**

Assuming that she takes the new job, Natasha would be willing to pay a risk premium equal to the difference between \$10,000 and the utility of the gamble so as to ensure that she obtains a level of utility equal to 3.162. We know the utility of the gamble is equal to 3.118. Substituting into her utility function we have,  $3.118 = I^{0.5}$ , and solving for  $I$  we find the income associated with the gamble to be \$9,722. Thus, Natasha would be willing to pay for insurance equal to the risk premium,  $\$10,000 - \$9,722 = \$278$ .