ECONOMICS 7330, Fall 2024
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Midterm Exam 2, October 21, 5 questions. All sub-questions carry equal weight.

1. (20%) Consider a sample of N independent binomially distributed random variables x_i taking values 0, 1, or 2 with probabilities

$$\pi(x) = \frac{2!}{x!(2-x)!} p^x (1-p)^{2-x} .$$

a) Find the ML-estimator for p.
b) If the sample is $x_i = 1, 2, 1, 0$,what is the estimate $\hat{p}$?

2. (16%) Given a sample of random variables x_i each with mean μ. Prove that if $\sqrt{N} Var(\overline{x})$ converges to a finite variance σ^2, then $\overline{x}$ converges in probability to μ.

3. (20%) Assume that $\sqrt{N}(\overline{x} - \mu)$ converges in distribution to $N(0, \sigma^2)$. Find the joint asymptotic distribution of $\sqrt{N}(\overline{x}^2 - \mu^2)$ and $\sqrt{N}(\frac{1}{\overline{x}} - \frac{1}{\mu})$ (the main part of the answer is a 2×2 variance matrix).

4. (24%) Assume that $(X, Y, Z)'$ is a vector normally distributed random variable with $\mu' = (1, 2, 3)$ and variance-covariance matrix

$$\Sigma = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} .$$

a) What is the conditional mean $E(X|Y)$?
b) What is the conditional mean $E(X|Z)$?
c) What is the conditional mean $E(X|Y, Z)$?
d) What the variance $Var(X|Y)$?
e) What the variance $Var(X|Y, Z)$?
f) What the variance $Var(X, Y|Z)$?

5. (20%) Let

$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix}$$

be the variance matrix of a vector X where $X = (X_1 , X_2)'$.

i) Find real numbers a, b and c such that $Y_1 = a\,X_1$ and $Y_2 = b\,X_1 + c\,X_2$ are uncorrelated and each have variance 1.
ii) Explain how the numbers that you found in part i) gives you a version of $\Sigma^{-1/2}$. (If you didn't solve part i), you can still get full points if you explain what is going on carefully.)