ECONOMICS 7330, Fall 2024
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Midterm Exam 1 Monday Sep 23 — 5 questions. All sub-questions carry equal weight except if otherwise indicated.

1. (21%) Consider a uniform distribution on $[0, 6]$.
a) Write down the CDF (distribution function). For all distributions, be specific about the support.
b) Find the 90th percentile.
c) What is the density function for Y if $Y = X^2$ when X follows the uniform distribution of the previous sub-questions.

2. (21%)
a) What is the formula for the marginal density $f_X(x)$, when you are given the joint density $f(x, y)$?
b) Let $f(x, y) = \frac{1}{2\sqrt{2\pi}} e^{-0.5[x^2+y]}$ be the joint density function for some random variables X and Y, where X takes values on the real line and Y on the positive real line. Find the marginal densities $f_X(x)$ and $f_Y(y)$.
c) What is the probability that $X < 0$ and $Y < 5$? (The question is about the probability of the joint event.)

3. (42%) Consider two random variables X and Y. Assume they both are discrete and that X can take the values 0, 1, and 2 while Y can take the values 0, 2, and 3. The probabilities for (X,Y) are shown in the following table:

	X=0	X=1	X=2
Y=0	2/15	1/15	2/15
Y=2	2/15	2/15	0/15
Y=3	2/15	3/15	1/15

i) Find the mean and the variance of X.
ii) Are the random variables X and Y independent?
iii) Find the conditional distribution of Y given $X = 1$.
iv) Verify for these numbers that $EY = EE(Y|X)$.
v) Find the conditional variance $Var(Y|X = 0)$.
vi) Show the $Var(Y) = EVar(Y|X) + Var(E(Y|X)$. You can choose to prove the identity instead.

4. (6%) Assume X follows an exponential distribution with mean 3. Write down the density for X conditional on $X > 10$.

5. (10%) X and Y are standard normal variables with mean 0, variance 2, and correlation 0.5. Find the number α such that the random variable $Z = Y - \alpha X$ is distributed independently of X.