

Midterm Exam 1 Monday Sep 25 — 5 questions. All sub-questions carry equal weight except if otherwise indicated.

1. (20%) Consider a standard exponential distribution with density e^{-x} for $x > 0$.
 - a) Find $P(X < 1)$.
 - b) Find the 10% upper percentile for X .
 - c) What is the density function for Y if $Y = 1 - e^{-X}$? (Make clear the support.)
 - d) Now assume that you are told that $Y < 0.5$. Given that, what is $E(X)$ that is, $E(X|e^{-X} < 0.5)$?

2. (20%) X is exponentially distributed with mean 1 and Y is exponentially distributed with mean 2.
 - a) Write down the pdf's and cdf's for X and Y .
 - b) Derive the density of $Z = X + Y$.

3. (25%)
 - a) What is the formula for the marginal density $f_X(x)$, when you are given the joint density $f(x, y)$?
 - b) Let $f(x, y) = \frac{1}{2\pi}e^{-0.5[(x+y)^2 - 2xy - 2x + 1]}$ be the joint density function for some random variables X and Y taken values on the real line. Find the marginal densities $f_X(x)$ and $f_Y(y)$.
 - c) Are the two random variables in sub-question b) independent? (Why or why not, a correct answer with no argument does not give any points.)

4. (11%) For a vector random variables X and Y of the same dimension find $\text{Var}(AX + BY)$, where A and B are matrices of identical dimensions.

5. (24%) Consider two random variables X and Y . Assume they both are discrete and that X can take the values 0 and 2 while Y can take the values 1, 2, and 3. The probabilities for (X, Y) are shown in the following table:

	X=0	X=2
Y=1	2/15	4/15
Y=2	2/15	1/15
Y=3	2/15	4/15

- i) Find the mean and the variance of X .
- ii) Are the random variables X and Y independent?
- iii) Find the conditional distribution of Y given $X = 0$.
- iv) Verify for these numbers that $EY = EE(Y|X)$.