ECONOMICS 7330, Fall 2022
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Midterm Exam 1 — 5 questions. All sub-questions carry equal weight except if otherwise indicated.

1. (20%) Consider a uniform distribution on the closed interval [1 , 4]. Assume a random variable X follows this distribution.
a) What is the Cumulative Density Function (CDF)? (A correct answer needs to also specify the support.)
b) What is the density function (PDF)?
c) Find the mean of X. (You have to derive it, no points for just stating the value.)
d) Find the variance of X.

2. (20%) A study of college students finds that while 80 percent of college students are male, only 40 percent of college students with an A average are male. It is also found that 15 percent of female students have an A average. Assuming these results are accurate and reflect probabilities, answer the following questions.
a) Are "being a male student" and "having an A average" independent? Why or why not?
b) What is the probability that a randomly selected student has an A average?
c) What is the probability that a randomly selected male student has an A average? (If you did not get part b), you can get full points by assuming a value for the solution to b) and then doing the remainder correctly.)

3. (20%) Assume that X follows a standard exponential distribution with density e^{-x} for $x > 0$.

a) What is the density function for Y if $Y = 2X$?
b) Find $P(X < 1)$.
c) Find the 10% upper percentile for Y.
d) Now assume that you are told that $X < 2$. Given that, what is $P(X < 1)$?

4. (20%)
a) What is the formula for the marginal density $f_X(x)$, when you are given the joint density $f(x, y)$?
b) Let $f(x, y) = (3/16)\, xy^2$; $0 < x < 2$, $0 < y < 2$, be the joint density function for some random variables X and Y. Find the marginal densities $f_X(x)$ and $f_Y(y)$.
c) Are the two random variables in sub-question b) independent? (Why or why not, a correct answer with no argument does not give any points.)

5. (20%) For a random variable X, a constant c, and two functions $g()$ and $h()$ prove that

a) (5%) $E[g(X) + c\, h(X)] \;=\; E[g(X)] \;+\; c\, E[h(X)]$.
b) (5%) Prove that $Var[cX] = c^2 Var(X)$.
c) (10%) Prove that the variance is always positive (zero if the random variable is a constant, but we assume it is not).