ECONOMETRICS I, SPRING 2024.

Homework 7. Due Wednesday March 20.

1.
a) Let

$$x_t \ = \alpha_0 \ + \ 5 * u_t \ + \ u_{t-1} \ ,$$

where u_t is white noise.
Find the autocovariances for x_t in terms of σ_u^2 (the variance of u_t).

b) Given the stationary AR(1) process

$$x_t \ = \ 3 \ + \ .5 * x_{t-1} \ + \ u_t$$

where $Eu_t^2 = 3$. Find the variance of x_t, and the first 3 autocovariances and autocorrelations.

2. For the AR(1) process:

$$e_t \ = \ a e_{t-1} \ + \ u_t \ ,$$

for $t = 1, 2, 3$ and $a = .6$.
a) Find the variance matrix $\Omega = var(e)$.
b) Find $\Omega^{-1/2}$ using the Prais-Winsten tranformation.
c) Verify by matrix multiplication that $\Omega^{-1/2} \, \Omega \, \Omega^{-1/2'} = I$.

3. Computer question (continuation of previous homeworks). In Matlab, regress real per capita U.S. data consumption growth on income growth and the interest rate using the posted dataset. (This is the what you did in homework 1.)

a) Calculate the residuals e. Regress e_t on e_{t-1}? Is there evidence of autocorrelation (Use t-tests.)
b) Assume you concluded that there is autocorrelation in the residuals (so don't condition on part a). Perform 2-stage GLS using the Prais-Winsten transformation.
c) Do the approximate feasible GLS estimation using the Cochrane-Orcutt transformation. Are you results sensitive to whether you do Prais-Winsten or Cochrane-Orcutt?