ECONOMETRICS I, SPRING 2018

Homework 5. Due Wednesday March 7.

1. Show, under the standard assumptions of the linear model (normality is not needed), that if you estimate a parameter $\tilde{\beta}$ under a set of linear restrictions, $R\beta = q$, the restricted estimator $\tilde{\beta}$ has lower variance that the unrestricted estimator $\hat{\beta}$. (Hint: Find the variance of $\hat{\beta} = \tilde{\beta} + (\hat{\beta} - \tilde{\beta})$— the key is that the covariance of $\tilde{\beta}$ and $\hat{\beta} - \tilde{\beta}$ is 0 [a matrix of zero's].)

2. Computer question (Monte Carlo). Using Matlab, you will examine how the OLS estimator performs as a function of the sample size and the distribution of the error terms in the regressors. You will use the normal random number generator to generate the error terms, the regressors, and the dependent variable. First, for N=500 draw a vector w of normal errors and construct $X_{1i} = 2 + w_i^2$. Draw another vector v of (standard) normal errors and construct $X_{2i} = 2 + w_i^2 + 4 * \log(v_i^2)$. There is no particular logic to the regressors except I make them correlated and not mean 0.

a) Now set N=20, Also draw a vector e of normal errors with variance 3 and construct $Y = \iota + 2X_1 + 4x_2 + e$ (this means that you only use the first 20 observations of the X's). Estimate $\hat{\beta}$ and construct the t-test for $\beta_1 = 2$ and save it. You do this 100 times and count how often the t-stat exceed the critical value for a 5% two-sided test (using the t-distribution with N-3 degrees of freedom).

Next we will try and figure out if the asymptotic distribution of the t-stat is a good approximation for N=20 and N=200. (You can try other values, when you have the loops set up, you should be able to just change one character to re-run the program).

b) Again set N=20. Draw a vector of standard normal errors and construct $z = e^2 - 1$ (the is a simple way to construct a mean zero error that is not normal). Construct $Y = \iota + 2X_1 + 4X_2 + z$. Estimate $\hat{\beta}_1$ and construct the asymptotic t-test for $\beta_1 = 2$ and save it. You do this 100 times and count how often the t-stat exceed the critical value for a 5% two-sided test (using the normal distribution).

c) Set N=200. Draw a vector of standard normal errors and construct $Z = e^2 - 1$ (the is a simple way to construct a mean zero error that is not normal). Construct $Y = \iota + 2X_1 + 4X_2 + z$. Estimate $\hat{\beta}_1$ and construct the asymptotic t-test for $\beta_1 = 2$ and save it. You do this 100 times and count how often the t-stat exceed the critical value for a 5% two-sided test (using the normal distribution). Does the asymptotic test perform better for the higher value of N?

3. Computer question (continuation of previous homeworks). In Matlab, regress real per capita U.S. data consumption growth on income growth and the interest rate using the posted dataset. (This is the what you did in homework 1.) Perform the following Chow tests (assuming the data

are normally distributed—even if you rejected this in the previous question).

a) Examine if the coefficients are the same before and after year 2008. (Use the appropriate test).

b) Examine if the coefficient to the interest rate is the same before and after year 2008 (assuming that the coefficient to income is not changing).

c) Examine if the coefficients are different in the last period.