ECONOMETRICS I, SPRING 2018

Homework 3. Due Wednesday February 21.

1. More Frisch-Waugh! Assume that your run a regression with 2 regressors (think of demeaned regressors). Assume the fitted value is

$$\hat{Y} = X_1 + 4X_2 \ .$$

Assume that you instead run the regression

$$(*) \quad Y = \gamma_1 \, X_1 + \gamma_2 \, M_1 X_2 + error \ ,$$

where M_1 is the residual maker from regressing on X_1. If P_1 is the projection matrix on X_1 and

$$P_1 X_2 = 1.5 X_1 \ ,$$

what would be the estimated values of γ_1 and γ_2 in the regression (*)?

2. For the bivariate Normal distribution, derive the formula for the conditional density $f(X_2|X_1)$.

3. (15% of Midterm 1, 2017) Assume that you want to estimate the following model using quarterly data for 10 years:

$$y_t = \beta_0 \ + \ \sum_{k=1}^{3} \beta_k D_{kt} \ + \ \beta_4 x_t \ + \ \epsilon_t \ ,$$

where all the "OLS-assumptions" - including normality of ϵ_t - hold. The regressors D_{kt} are quarterly dummy variables, such that

$$\begin{aligned} D_{1t} &= 1 \ \text{ in the 2nd quarter ; 0 otherwise} \\ D_{2t} &= 1 \ \text{ in the 3rd quarter ; 0 otherwise} \\ D_{3t} &= 1 \ \text{ in the 4th quarter ; 0 otherwise} \end{aligned}$$

Now assume that $\bar{y} = 5$ and if we let $\bar{y}_j \ ; \ j = 2, 3, 4$ denote the average of the y-values in the kth quarter, assume that

$$\begin{aligned} \bar{y}_2 &= 4 \ , \\ \bar{y}_3 &= 2 \ , \\ \bar{y}_4 &= 0 \ . \end{aligned}$$

Also assume that $\bar{x} = 0$ and that x_t is orthogonal to $D_k \ ; \quad k = 1, 2, 3$.

Based on the given information, find the values of the OLS-estimates $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$ and $\hat{\beta}_3$.

For the computer questions below, you may want to get some pointers from Eris.

3. Computer question (continuation of homeworks 1 and 2). In Matlab, regress real per capita U.S. data consumption growth on income growth and the interest rate using the posted dataset. (This is the what you did in homework 1.)

a) Calculate the residual maker M and (using Matlab) calculate and display the eigenvalues and eigenvectors of M.

b) Generate the C matrix and the diagonal matrix of eigenvalues Λ (in the notation of class) and verify that $C\Lambda C' = M$. Display the values of C and Λ.

4. Computer question.
a) Generate two vectors of standard normally distributed variables e_1 and e_2 of length $N = 100$.
b) Generate $X_1 = e_1$ and let $X_2 = e_1 + e_2$ and calculate the variance-covariance matrix Σ for $X = (X1, X2)$. (You can do that by hand, of course, but you will need to use it in the next question.)
c) Find a square root $\Sigma^{1/2}$ of Σ using Matlab.
d) Calculate $Y = (Y_1, Y_2)$ as $Y = \Sigma^{-1/2}X$.
e) Calculate the covariance between Y_1 and Y_2 and verify that it is (close to) zero.