

Homework 3. Due Wednesday February 15.

1. More Frisch-Waugh! Assume that you run a regression with 2 regressors (think of demeaned regressors). Assume the fitted value is

$$\hat{Y} = 3X_1 + 4X_2 .$$

Assume that you instead run the regression

$$(*) Y = \gamma_1 X_1 + \gamma_2 M_1 X_2 + error ,$$

where M_1 is the residual maker from regressing on X_1 . If P_1 is the projection matrix on X_1 and

$$P_1 X_2 = 1.5X_1 ,$$

what would be the estimated values of γ_1 and γ_2 in the regression (*)?

2. For the bivariate Normal distribution, derive the formula for the conditional density $f(X_2|X_1)$.

For the computer questions below, you may want to get some pointers from Xavier.

3. Computer question (continuation of homework 1 and 2). In Matlab, regress real per capita U.S. data consumption growth on income growth and the interest rate using the posted dataset. (This is the what you did in homework 1.)

a) Calculate the residual maker M and (using Matlab) calculate and display the eigenvalues and eigenvectors of M .

b) Generate the C matrix and the diagonal matrix of eigenvalues Λ (in the notation of class) and verify that $C\Lambda C' = M$. Display the values of C and Λ .

4. Computer question.

a) Generate two vectors of standard normally distributed variables e_1 and e_2 of length $N = 100$.

b) Generate $X_1 = e_1$ and let $X_2 = e_1 + e_2$ and calculate the variance-covariance matrix Σ for $X = (X_1, X_2)$. (You can do that by hand, of course, but you will need to use it in the next question.)

c) Find a square root $\Sigma^{1/2}$ of Σ using Matlab.

d) Calculate $Y = (Y_1, Y_2)$ as $Y = \Sigma^{-1/2} X$.

e) Calculate the covariance between Y_1 and Y_2 and verify that it is (close to) zero.