ECONOMETRICS II, FALL 2023

Homework 4. Due Wednesday September 20.

1. Use the posted Matlab `Main_HW4.mlx` program. The simulation part is set up to generate a model with "general selectivity" as covered in class. Show that the ML estimation of the standard linear model is biased in the face of selection. (This means that OLS is biased as well.) This code is posted.

2. Program up the correct ML estimator and check that the estimates from the linear part are less biased. (We know that the ML estimator in general is consistent, and usually not too biased, but I don't know what the exact bias is, your simulation will show.)

3. Consider the model $y_t^0 = X_t\beta + u_t$ where u_t is a Normally distributed with variance σ^2. Assume that there is time varying censoring such that you observe $y_t = y_t^0$ if $y_t^0 \leq K_t$ and $y_t = K_t$ if $y_t^0 \geq K_t$ for some time varying variable K_t. Write down a consistent estimator of β.

If K_t is an observed random variable, uncorrelated with u_t what, if anything, would you change to get a consistent estimator of β?

4. (25% of 2018 Final. For the homework, you have access to the book and notes, but please explain this in your own words without looking at those. For you own sake.)
a) (10%) Explain what is meant by a censored and by a truncated regression model and explain why a simple OLS-estimate is biased (explain the direction of the bias–it depends on the true slope and on whether the truncation is at the top or the bottom). Figures are fine.

b) (15%) Assume you are estimating the model

$$Y_i = aX_i + u_i \ ,$$

by OLS. Here a is a scalar and we assume for simplicity that there is no intercept and that in the true underlying model (not censured or truncated) the error term has mean 0.
Assume that you only have 3 observations: $X' = 2, 4, 6$ and that the data are truncated such that X, Y is dropped for values of Y larger than 6. Also assume that we know that the distribution of the innovation term U is such that it takes only the values -2 and 2 (each with probability 0.5) and that the true value of a is 1.
Find the expected value of the OLS estimator of a (hint: for each value of X find the expected value of the truncated error).