

**ECONOMICS 6331 – Probability and Statistics, Fall 2022**

Homework 7. Due Wednesday October 19.

1. Assume that  $X$  and  $Y$  follows a bivariate normal distribution.
  - a) Show that  $X - E(X|Y)$  is independent of  $Y$ . (Use the law of iterated expectations or just find the covariance.)
  - b) Find the variance of  $X - E(X|Y)$  (hint: This a linear function of  $X$  and  $Y$ ).
  - c) Now show that  $\frac{1}{(1-\rho^2)}[(\frac{X-\mu_X}{\sigma_X})^2 - 2\rho(\frac{X-\mu_X}{\sigma_X})(\frac{Y-\mu_Y}{\sigma_Y}) + (\frac{Y-\mu_Y}{\sigma_Y})^2]$  is distributed as  $\chi^2(2)$ .
2. Using square roots of matrices show that if  $X \sim N(0, \Sigma)$  where  $X$  is  $k$ -dimensional, then  $X'\Sigma^{-1}X$  is  $\chi^2$  distributed with  $k$  degrees of freedom.
3. (12% of 2003 final) Assume  $X \sim N(0, 9)$ ,  $Y \sim N(2, 9)$ , and  $Z \sim N(2, 16)$ . Further assume that the covariance between  $X$  and  $Y$  is 2, while both  $X$  and  $Y$  are independent of  $Z$ .
  - i) What is  $E(X|Y = 2, Z = 3)$ ? (State the formula you use and then the number.)
  - ii) What is the conditional variance  $Var(X|Z = 3)$ ?