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ECONOMICS 7330 – Probability and Statistics, Fall 2023

Homework 6. Due Wednesday October 11.

1. Assume that X, Y, and Z follows a normal distributions.
Denote the covariance between X and Y Σ_{XY} and the variance of X σ_X^2 and similarly for the other variances covariances.
a) Write down the joint density of X and Y using scalars.
b) Find the conditional density $F(X, Y|Z)$ by dividing the density from part a) with the marginal density of Z.
c) Write down mean and variance of X, Y in vector/matrix form (the variance matrix is 2 by 2, for example). Write down the density in vector-matrix notation.
d) Use the matrix formulas for the conditional density of X, Y given Z to find the conditional distribution and verify that you get the same as you got in part b).

2. (12% of 2003 final) Assume $X \sim N(0, 9)$, $Y \sim N(2, 9)$, and $Z \sim N(2, 16)$. Further assume that the covariance between X and Y is 2, while both X and Y are independent of Z.
i) What is $E(X|Y = 2, Z = 3)$? (State the formula you use and then the number.)
ii) What is the conditional variance $Var(X|Z = 3)$?

3. Consider an i.i.d. sample $X_1, ..., X_N$. Define the residual $e_i - X_i - \overline{X}$. Verify that $\overline{X}e_i$=0.