

ECONOMICS 7330 – Probability and Statistics, Fall 2025

Homework 4. Due Wednesday October 8.

1. . Assume that X and Y follows a bivariate normal distribution. Demonstrate that $X + Y$ is normally distributed—a super important result. (Hint: use the convolution formula and “complete the square.” You can use that $(z - s)^2 + s^2 = 2 * (s - \frac{z}{2})^2 + \frac{z^2}{2}$.)
2. Assume that X and Y are exponentially distributed, independently and with mean θ . Find the distribution of $X + Y$.
3. Assume that X is an n -dimensional random variable with covariance matrix Σ and Y is an n -dimensional random variable, independent of X with covariance matrix Ω . Show that the covariance matrix for $X + Y$ is $\Sigma + \Omega$.
4. (24% of final 2005) Assume that Z is a normally distributed random variable with variance 9 and mean 2, and that Z is independent of (X, Y) where (X, Y) is a bivariate normally distributed random variable with mean $\mu' = (0, 0)$ and variance-covariance matrix

$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

- a) What is the conditional mean of $Y|X$?
- b) What is the conditional variance of (X, Z) given Y ?
- c) What is the conditional mean of X given (Y, Z) ?
- d) What is the distribution of $2X^2 - 2XY + Y^2$? (This paranthesis was not on the exam, but show that it is chi-square with two degrees of freedom, also I will show on Wednesday how to get from Normal to chi-square. This is also a super important result.)