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ECONOMICS 7330 – Probability and Statistics, Fall 2023

Homework 4. Due Wednesday September 20.

1. For a random variable X with uniform distribution on the interval $[a, b]$ with density f show
(a) $\int_a^b f(x)dx = 1$.
(b) $E[X] = \frac{1}{2}(b-a)$.
(c) $var[X] = \frac{1}{12}(b-a)^2$.

2. Show that if X and Y are statistically independent, and $a, b, , c, d$ are real constants with $a < b$ and $c < d$, then

$$P[a < X < b, c < Y < d] = P[a < X < b]P[c < Y < d] .$$

3. Prove that for any random variables X and Y with finite variances (hint: use the law of iterated expectations):
(a) The covariance $cov(X, Y) = cov(X, E[Y|X])$.
(b) X and $Y - E[Y|X]$ are uncorrelated. (This implies they are independent if they are normally distributed. This is sometimes important.)

4. Suppose that Y conditional on X is $N(X, X)$ (that is, Normally distributed with both mean and variance equal to X). If $E[X] = \mu$ and $var(X) = \sigma^2$ what are $E[Y]$ and $var[Y]$? (hint: use the law of iterated expectations.)

5. Consider two random variables X and Y. Assume they both are discrete and that X can take the values 1,2, and 4 while Y takes the values 0 and 2. The probabilities for (X,Y) are shown in the following table:

	X=1	X=2	X=4
Y=0	3/24	3/24	6/24
Y=2	3/24	5/24	4/24

i) Find the marginal probabilities of X and Y. Mark clearly which are the marginal probabilities of X and which are the marginal probabilities of Y. Explain what the marginal probabilities measure.
ii) Find the means and the variances of X and Y.

iii) Are the events X $= 1$ and Y $= 2$ independent events?
iv) Are the random variables X and Y independent?
v) Find the probability $P(\{X > 1\} \bigcap \{Y \leq 1\})$
vi) Find the conditional distribution of X given $Y = 2$.
vii) Find the random variable $E(X|Y)$.
viii) Take the mean of the random variable that you derived in vii) and verify that it equals $E(X)$.