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ECONOMICS 7330 – Probability and Statistics, Fall 2025

Homework 3. Due Wednesday October 1.

1. Prove that for any random variables X and Y with finite variances (hint: use the law of iterated expectations) X and $Y - E[Y|X]$ are uncorrelated. (This implies they are independent if they are normally distributed. This result is important.)

2. Consider two random variables X and Y. Assume they both are discrete and that X can take the values 1,2, and 4 while Y takes the values 0 and 2. The probabilities for (X,Y) are shown in the following table:

	X=1	X=2	X=4
Y=0	3/24	3/24	6/24
Y=2	3/24	5/24	4/24

i) Find the marginal probabilities of X and Y. Mark clearly which are the marginal probabilities of X and which are the marginal probabilities of Y. Explain what the marginal probabilities measure.
ii) Find the means and the variances of X and Y.
iii) Are the events $\text{X} = 1$ and $\text{Y} = 2$ independent events?
iv) Are the random variables X and Y independent?
v) Find the probability $P(\{X > 1\} \bigcap \{Y \leq 1\})$
vi) Find the conditional distribution of X given $Y = 2$.
vii) Find the random variable $E(X|Y)$.
viii) Take the mean of the random variable that you derived in vii) and verify that it equals $E(X)$.
xi) Find the probability distribution for $Z = X + Y$.