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ECONOMICS 7330 – Probability and Statistics, Fall 2023

Homework 3. Due Wednesday September 13.

1. Let $X \sim U[0,1]$ be uniformly distributed on $[0,1]$. Suppose X is truncated to satisfy $X \leq c$ for some $0 \leq c \leq 1$.
(a) Find the density function of the truncated variable X.
(b) Find $E[X|X \leq c]$.

2. (a) Show that if X is uniformly distributed on the interval $[0,1]$ then $Y = -\theta \log(X)$ follows an exponential distribution with mean θ.
(b) Explain why Jensen's inequality implies that $E(Y) > \log(2)$ for $\theta = 1$.

3. Let $f(x,y) = (3/16)\, xy^2$; $0 < x < 2$, $0 < y < 2$, be the joint density function for X and Y. Find the marginal density functions $f_X(x)$ and $f_Y(y)$. Find the distribution function (CDF) for X. Are the two random variables independent?

4. Let the joint probability function for X and Y be defined by

$$f(x,y) = \frac{x+y}{32} \ , \quad x = 1,2; \ y = 1,2,3,4 \,.$$

Find
a) $f_X(x)$, the marginal probability function for X.
b) $f_Y(y)$, the marginal probability function for Y.
c) $P(X < Y)$.
d) $P(Y = 3X)$.
e) $P(X + Y = 4)$.
f) $P(X \leq 4 - Y)$.
g) Are X and Y independent or dependent?