ECONOMICS 7330—Probability and Statistics, Fall 2025

Homework 2. Due Wednesday September 24.

1. Show that if the density satisfies $f(x) = f(-x)$ for all $x \in R$ then the distribution function satisfies $F(-x) = 1 - F(x)$.

2. Let $X \sim U[0,1]$ be uniformly distributed on $[0,1]$. Suppose X is truncated to satisfy $X \leq c$ for some $0 \leq c \leq 1$.
(a) Find the density function of the truncated variable X.
(b) Find $E[X|X \leq c]$.

3. (a) Show that if X is uniformly distributed on the interval $[0,1]$ then $Y = -\theta \log(X)$ follows an exponential distribution with mean θ. (You need to explicit about the support of the variables.)

4. Let $f(x,y) = (3/16)\, xy^2$; $0 < x < 2$, $0 < y < 2$, be the joint density function for X and Y.
(a) Find the marginal density functions $f_X(x)$ and $f_Y(y)$.
(b) Find the distribution function (CDF) for X.
(c) Are the two random variables independent?

5. Let the joint probability function for X and Y be defined by

$$f(x,y) \;=\; \frac{x+y}{32}\;,\; x = 1,2\,;\; y = 1,2,3,4\,. \quad \text{Find:}$$

a) $f_X(x)$, the marginal probability function for X.
b) $f_Y(y)$, the marginal probability function for Y.
c) $P(X < Y)$.
d) $P(Y = 3X)$.
e) Are X and Y independent or dependent?