Bent E. Sørensen

ECONOMICS 7330—Probability and Statistics, Fall 2025

Homework 1. Due Wednesday September 17.

1. Prove that $P[A \bigcup B] \geq P[A] + P[B] - 1$.

2. Prove that $[\bigcup_{i=1}^{\infty} A_i]^C = \bigcap_{i=1}^{\infty} A_i^C$.

3. (Hansen 1.18). A, B, and C are three events. If $P(A \bigcap B|C) = P(A|C) * P(B|C)$ we say that A and B are conditionally independent given C. Consider the experiment of tossing two dice. Let A={First die is 6}, B={Second die is 6}, and C={The two dies are identical}. Show that A and B are (unconditionally) independent, but A and B are dependent given C.

4. We observe the price of 2 stocks, stock A and stock B.
The probability that the price of stock A increases is 0.7 and the probability that the price of stock B increases is 0.5. We also know that the probability of the event that the price of either stock A or stock B increases is 0.9.
a) What is the probability that the price of stock A increases at the same time as the price of stock B increases?
b) What is the probability that the price of A increases if you observe that the price of stock B increases?
(State clearly which formulas you used).

5. Let B be an event and $A_1, A_2, ..., A_n$ be n mutually exclusive events. Define $A = \bigcup_{i=1}^{n} A_i$. Also assume $P(A_i) > 0$ and $P(B|A_i) = p$ for all i. Show that $P(B|A)$ is also equal to p. [A Venn diagram might help.]

6. Let $X \sim U[0,1]$ (uniform distribution). Find the PDF (density) of $Y = X^2$. (Use the formula, or find the CDF first.)

7. Let $X \sim U[0,1]$. Find the distribution function of $Y = \log(\frac{X}{1-X})$.