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ECONOMICS 7330 – Probability and Statistics, Fall 2023

Homework 10. Due Wednesday November 15.

1. Consider the exponential model with density

$$f(t_i) = \frac{1}{\lambda_i} \exp(-t_i/\lambda_i)\,,$$

where λ_i is the mean (in a major application of the exponential distribution it is the expected waiting time for individual i). We cannot estimate a parameter for each person, but we can estimate the effect of covariates by assuming $\lambda_i = x_i\beta$, where we now have a limited number of parameters as the dimension of β is the number of covariates ("regressors") included.

Find the Score and Hessian (or what Hansen calls "Likelihood Hessian" which is multiplied by –1). (Note, the parameter is now β. In general, β is a vector, but you could start by solving the univariate case.)

2. Consider the normal model, with its mean a function of individual-specific covariates (regressors). Here there is just one regressor. The ML-estimator maximizes the log-likelihood (suppressing the π term that does not affect maximum:

$$\Sigma_{i=1}^{N} - 0.5\log(\sigma^2) - 0.5\frac{(y_i - \beta x_i)^2}{\sigma^2}\,.$$

The parameter vector now is $\theta' = (\beta, \sigma^2)$.

i) Find the ML estimator. (Of β and σ^2. We already found $\hat{\beta}$ in the handout.)
ii) Find the score (a 2-dimensional vector).
iii) Find the Hessian and its expected value.
iv) Show that the information equality holds.

3. (Hansen Exercise 13.2.) Take the Poisson model with parameter λ. We want a test for $H_0 : \lambda = 1$ against $H_1 : \lambda \neq= 1$. (a) Develop a test based on the sample mean X_n. (b) Find the likelihood ratio statistic

4. (Hansen Exercise 13.8.) This is an easy question, but good to think about.) You

teach a section of undergraduate statistics for economists with 100 students. You give the students an assignment: They are to find a creative variable (for example snowfall in Wisconsin), calculate the correlation of their selected variable with stock price returns, and test the hypothesis that the correlation is zero. Assuming that each of the 100 students selects a variable which is truly unrelated to stock price returns, how many of the 100 students do you expect to obtain a p-value which is significant at the 5% level? Consequently, how should we interpret their results? (In economics, and no doubt many other disciplines, publication bias is a big problem. A top journal gets 100 submissions but only accepts those with significant results. Because of this selection, a large fraction of the significant results published may in fact be spurious.)

5.