ECONOMICS 7330, Fall 2024
Bent E. Sørensen

Final Exam, December 2, 2024—6 questions (100 points). All sub-questions carry equal weight unless noted.

1. (40%) Consider two random variables X and Y. Assume they both are discrete and that X can take the values 0 and 1, while Y can take the values 0, 1, and 3. The probabilities for (X,Y) are shown in the following table:

	X=0	X=1
Y=0	2/15	4/15
Y=1	1/15	2/15
Y=3	2/15	4/15

i) Find the marginal probabilities of X.
ii) Find the mean and the variance of X.
iii) Are the events X = 0 and Y = 1 independent events?
iv) Are the random variables X and Y independent?
v) Find the conditional distribution of y given $X = 0$ and the conditional distribution of y given $X = 1$.
vi) Verify that $EE(Y|X) = EY$. Alternatively, prove the general formula.
vii) Find $Var(Y|X = 0$ and $Var(Y|X = 1)$.
viii) Verify that $Var(Y) = E(Var(Y|X)) + Var(E(Y|X)$. Alternatively, prove the general formula.

2. (10%) Consider an exponentially distributed random variable X with mean 3.
a) Find the 10% upper percentile for X.
b) Now assume that you are told that $X < 4$ (a truncated distribution). Given that, what is $E(X)$ that is, $E(X|X < 4)$?

3. (10%) Consider a sample of N independent bivariate normally distributed random variables $W_i = (X_i, Y_i)'$ with density

$$\frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{(x_i-\mu_x)^2+(y_i-\mu_y)^2-2\rho(x_i-\mu_x)(y_i-\mu_y)}{2(1-\rho^2)}}.$$

Here I have set the variance of x and y to unity to get a simpler expression and ρ the correlation which we assume known.
Find the ML-estimator for $(\mu_x, \mu_y)'$.

4. (10%) Assume that $Z = (Z_1, Z_2, Z_3, Z_4)'$ is a vector normally distributed random variable with mean μ and variance-covariance matrix Σ, where

$$\Sigma = \begin{pmatrix} 4 & 0 & 1 & 0 \\ 0 & 4 & 1 & 1 \\ 1 & 1 & 6 & 2 \\ 0 & 1 & 2 & 4 \end{pmatrix} \quad \text{and} \quad \mu = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

a) What is the conditional mean of $(Z_1 + Z_3)$ given Z_2?
b) What is the conditional mean of $(Z_1 + Z_3)$ given $(Z_1, Z_2)'$?

5. (15%) Consider a random sample of men and women. 40% of the sample are women. The probability that a man is pro free trade is 30% and the probability a woman is pro free trade is 50%. Some of the the individuals in the sample are economists. (The probability that an economist if pro free trade is, of course, 100%). 10% of the sample are female economists and 10% of the sample are male economists.
1) Assume you select a woman. What is the probability that she is an economist?
2) Again select a woman. You find out that she is pro free trade. What is the probability that she is an economist?
3) Assume you draw (pick) 5 individuals according to these probabilities (each draw has the same probability and the draws are independent). What is the expected number of women?

6. (15%) Assume that $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)'$ converges in distribution to a vector normally distributed random variable with mean 0 and variance-covariance matrix Σ.

$$\Sigma = \begin{pmatrix} 4 & 2 \\ 2 & 9 \end{pmatrix}$$

a) What is the asymptotic distribution of $\sqrt{N}(\hat{\theta}_1 - \hat{\theta}_2)$? (It is implicit here that $\hat{\theta}$ depend on N and N goes to infinity.)
b) If you have 90 observations and rely on the asymptotic distribution as an approximation. What would be the standard error that you would use for $\hat{\theta}_1$?
c) What is the asymptotic distribution of $\sqrt{N}\hat{\theta}_2^3$? (Explain which rule allows you to make your conclusion.)