ECONOMICS 7330, Fall 2023
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Final Exam, December 4, 2023—6 questions (100 points). All sub-questions carry equal weight unless noted.

1. (16%) Imagine that we select adults from either Texas (TX) or California (CA). Each person can have bachelor degree (BA) or not. Use the following (made up) probabilities:
The probability that a Californian has a BA is 60%, and the probability that a Texan has a BA is 30%.
Now assume you examine 5 people from CA and 4 people from TX (all independently sampled).

a) What is the expected number of BAs?
b) If X is the number of BAs in the sample. What is the variance, $\text{var}(X)$?
c) If we randomly pick someone with a BA from the sample, what is the probability that this person is from Texas?
d) If you randomly pick a person from the sample, what is the probability that this person is Texan with a BA?

2. (12%) Consider a uniformly distributed random variable X on the interval from 2 to 5.
a) Find $P(X < 4)$.
b) Find the 10% upper percentile for X.
c) Now assume that you are told that $X < 4$ (a truncated distribution). Given that, what is $E(X)$ that is, $E(X|X < 4)$?

3. (12%) Consider two random variables X and Y. Assume they both are discrete and that X can take the values 0 and 2 while Y can take the values 1 and 4. The probabilities for (X,Y) are shown in the following table:

	X=0	X=2
Y=1	2/15	3/15
Y=4	1/15	9/15

i) Find the mean and the variance of X.
ii) Are the random variables X and Y independent?
iii) Verify for these numbers that $EX = EE(X|Y)$. (Hint: first show the distribution of the random variable $E(X|Y)$.)

4. (20%) Consider a sample of N independent log-normally distributed random variables x_i with density $\frac{1}{\sqrt{2\pi}x\sigma}e^{-0.5(\ln(x)-\mu)^2/\sigma^2}$.
a) Find the ML-estimator for μ.
b) Write down an expression for the variance of the ML-estimator for μ. (You can assume σ known for this sub-question.)

5. (20%) Assume that $Z = (Z_1, Z_2, Z_3, Z_4)'$ is a vector normally distributed random variable with mean μ and variance-covariance matrix Σ, where

$$\Sigma = \begin{pmatrix} 4 & 0 & 1 & 0 \\ 0 & 4 & 1 & 1 \\ 1 & 1 & 6 & 2 \\ 0 & 1 & 2 & 4 \end{pmatrix} \quad \text{and} \quad \mu = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

a) What is the conditional mean of Z_1 given Z_2?
b) What is the conditional mean of (Z_1, Z_3) given Z_2?
c) What is the conditional mean of $(Z_1 + Z_3)$ given Z_2?
d) What is the conditional mean of (Z_1, Z_3) given (Z_2, Z_4)?

6. (20%) Assume that $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ converges in distribution to a vector normally distributed random variable with mean 0 and variance-covariance matrix Σ, where

$$\Sigma = \begin{pmatrix} 4 & 2 \\ 2 & 9 \end{pmatrix}$$

a) What is the asymptotic distribution of $\hat{\theta}_1 - \hat{\theta}_2$? (It is implicit here that $\hat{\theta}$ depend on N and N goes to infinity.)
b) What is the asymptotic distribution of $\frac{1}{9}\hat{\theta}_2^2$? (Explain which rules allows you to make your conclusion.)