ECONOMICS 7344, Spring 2020
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Final Exam, April 29—5 questions, total weight is 100%, all sub-questions carry equal weight except where noted.

1. (10%) A consumer lives for 3 periods (periods 1, 2, and 3), earns \$100 in the first period and in the third period. In period 2, income is either \$90 or \$110 with probabilities of 0.5. The consumer has a logarithmic utility function and is allowed to freely borrow and lend at an interest rate that equals his or her rate of time preference which we for simplicity set to 0 (i.e., the net rate of interest is 0).
A) Is $E_1(C_2) = C_1$? Explain why or why not.
B) Is $C_3 = C_2$?

2. (15%) Assume that income follows the ARMA process

$$y_t = 3 + 0.4 y_{t-1} + 0.2 y_{t-2} + e_t \,,$$

where e_t is white noise.
a) Is this time-series process stable?
b) What is $E_{t-2} y_t$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?
c) What is $E_{t-2} y_{t+1}$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?

3. (15%) Assume that Joe's wage income follows the AR(2) process

$$y_t = 3 + 0.9 y_{t-1} + 0.1 y_{t-2} + e_t \quad (*)$$

where e_t is white noise with variance 3.

Assume Joe's boss gave Joe a wage of 100\$ period 0 and a wage of 50\$ in period 1.
i) (5%) What is Joe's expected wages in periods 2 and 3?
ii) (10%) If Joe's boss gave Joe a raise of 50\$ in period 2, what would be Joe's change in consumption if the rate of interest is 10%. (Assume the PIH holds.)

4. (10%) Consider the CAPM-model. Assume the safe rate of interest is 3%, the mean return to the market portfolio is 5% and the variance of the return to the market portfolio is 0.02. Now consider assets D and E. For these we know the distribution of the pay-outs. For D the payout is normally distributed with mean 200 and variance 10, while for E the payout is normally distributed with mean 1000 and variance 5. Assume the covariance of the payout to asset D with the market return is 1 while the covariance of payout to asset E with the market return is 2.
What would be the prices of assets D and E?

5. (50%) Assume an economy consists of three agents (Jones, Smith, and Cooper) who each maximize a von Neumann-Morgenstern utility function

$$U(C_0) + E_0 U(C_1) \ ,$$

where $U(C_t) = \log C_t$. There are two time periods ($t = 0$ and $t = 1$), no storage, and two states-of-the-world. There are two time periods ($t = 0$ and $t = 1$), no storage, and two states of the world, "A" and "B," each having probability 0.5.

The following table gives the state-specific endowments for Jones, Smith, and Cooper (note that there is no uncertainty in period 0):

	Jones		Smith		Cooper	
State of the world:	A	B	A	B	A	B
period 0 endowment	50	50	50	50	50	50
period 1 endowment	25	75	75	25	50	100
Probability:	.50	.50	.50	.50	.50	.50

For the first three sub-questions assume Jones and Smith are the only agents in the world.

a) Assume the agents in period 0 can trade in a bond that matures in period 1 (equivalently, they can borrow from each other in period 0) but not in any other assets. Is the rate of interest positive or negative? (Make sure to state in economic terms why this is the intuitive answer. If you are pressed for time and explain logically why the interest rate must be negative or positive, you will get most of the points.)
b) Explain intuitively whether Smith will buy or sell bonds and why.

For the remaining questions assume that the agents have access to Arrow securities for both state A and state B.

c) Find the consumption of Jones in all states of the world and all periods.

From here on, assume Jones, Smith, and Cooper are the only agents in the world and keep assuming the agents have access to a full set of Arrow securities.

d) Find the prices of the Arrow securities and the interest rates.
e) Explain intuitively why the interest rate changes from the initial situation without Cooper (give at least two reasons).
f) Find the consumption of all three agents in all periods.
g) Explain intuitively why the ratio of Smith's consumption to Jones's consumption changes when Cooper is included.