ECONOMICS 7344, Spring 2020
Bent E. Sørensen

HOMEWORK 5. Due Thursday April 23.

1. (18% of make-up core exam August 2018) Consider the case of the 2 agents, Jones (J) and Smith (S) who live for 2 periods in a 3 states-of-the-world economy. Assume each of the agents have utility functions

$$\ln(C_0) + 0.9\, E_0 \ln(C_1) \ .$$

The following table gives the possible endowments and probabilities for Jones and Smith:

	Jones			Smith		
State of the world:	1	2	3	1	2	3
period 0 endowment	50	50	50	50	50	50
period 1 endowment	25	50	50	50	25	100
Probability:	.50	.25	.25	.50	.25	.25

Assume that Jones and Smith are the only two agents in the world.

(a) Assume Jones and Smith in period 0 can trade in a bond that matures in period 1 (equivalently, one can borrow from the other in period 0) but not in any other assets. What is the rate of interest? (If you state the two equations in two unknowns that determines the solution *and* states whether the interest rate is positive, 0, or negative, that is considered a full answer.)

(b) Now assume that J and S can trade in a bond *and* and Arrow security for state 1 (there are no Arrow securities for the other states of the world). Find the price of the Arrow security.

(c) Under the assumption of part b, find the rate of interest between period 0 and period 1. (For this question, it is sufficient to state the equations that would need to be solved, rather than finding the explicit solution.)

2. (15% of the June 2014 core exam) Consider the case of the 2 agents, 2 periods, 2 states-of-the-world model of Obstfeld-Rogoff Chapter 5.2 (where agents can trade using a full set of Arrow securities). Assume that both agents have utility functions $U(C_0) + E_0U(C_1)$, where $U(C_t) = -\exp(-C_t)$.

Assume that the endowment of the first agent is $y_0 = 3, y_1 = 3$ and that the endowment of the second agent in period 0 is $y_0^* = 3$ and in period 1 his or her endowment is $y_1^* = 6$ in the "good state" g. In the "bad state" b the endowment of the second agent is $y_1^* = 0$. Assume that the good state happens with probability 0.5.

a) Derive the formula for the rate of interest as a function of initial endowments and period 1 endowments.

b) Now assume that in period 1 the endowment of the second agent is $y_1^* = 4$ in the "good state" and $y_1^* = 2$ in the "bad state." Assume that the good state happens with probability 0.5. Will the rate of interest go up or down compared to the initial situation? (It is more important that you argue the logic than solving for numbers—you don't have to do numbers at all in the question if you argue clearly.)

3. (35% of 2018 final) Assume an economy consists of N agents who each maximize a von Neumann-Morgenstern utility function

$$U(C_0) + E_0U(C_1) ,$$

where, until further notice, $U(C_t) = -\frac{1}{\gamma}exp{-\gamma C_t}$ where γ is a positive constant. Assume there is no storage and perfect Arrow-Debreu markets. There are two time periods ($t = 0$ and $t = 1$) and two states of the world "A" and "B" in period 1. Assume there are N consumers in the economy.

a) Demonstrate (do the derivations) that this economy allows for a representative agent.

b) Derive a formula for the relation between the consumption of each agent and the aggregate consumption

Now assume that $N = 2$, $\gamma = 1$, and the endowment of the first agent is $y_0^1 = 3$, $y_1^1 = 5$ in state A and $y_1^1 = 1$ in state B. The endowment of the second agent in period 0 is $y_0^2 = 3$ and in period 1 his or her endowment is $y_1^2 = 1$ in state A and $y_1^2 = 5$ in state B. Assume that state A happens with probability 1/2.

c) For now assume that agents can trade in a bond but no other financial assets exist. Find the rate of interest.

d) Explain why the rate of interest is positive or negative using concepts from the class.

e) Now assume that the agents can trade in Arrow securities for state A and state B. Find the prices of the Arrow securities and the rate of interest. Explain why is it higher or lower than the rate of interest you found in question c).

f) Under the assumptions of part b), find the consumption of each agent in each period and in each state of the world.

4. (Use the note on market spanning.) Consider the case of an economy with four states-of-the-world. Assume that an asset S_1 exists that pays 2 units in period 1 if state A occurs, 1 unit if state B occurs, and nothing if state C or D occurs. Another asset S_2 exists which pays 1 unit in period 1, if state C occurs, and nothing in states A, B, and D. A third asset S_3 pays 0 units in period 1 if state C occurs, and 2 units in states A, B, and D. Finally, a discount bond paying one unit in period 1 for sure can be traded.

a) Is the set of assets equivalent to a full set of Arrow securities?

b) Now assume that asset S_3 instead pays 1 unit in period 1, if state A occurs, and 0 units in states B, C, and D. Are the markets perfect (equivalent to a full set of Arrow securities) in this case?