Bent E. Sørensen

ECONOMICS 7344 – MACRO II, part b, Spring 2020

Homework 2. March 25, due Wednesday April 1.

1. (From the January 2011, make-up core exam.) A consumer lives for 2 periods and earns $Y_1 = 20\$$, in period 1, and in period 2 he or she earns $Y_2^a = 20\$$ with probability $1/2$ (state a) and $Y_2^b = 30\$$ with probability $1/2$ (state b). The consumer starts with 0 assets and maximizes

$$U(C_1) + \frac{1}{1.10} E_1 U(C_2) \, ,$$

where

$$U(C) = 100C - \frac{1}{2} C^2 \, .$$

Assume that the safe rate of interest is 10 percent.

A) (5%) Let B denote the amount lent in period 1 (or, equivalently, the amount of a safe bond bought). Assuming that the agent only have access to a safe bond, find B and consumption in each period (for period 2, that means the consumption plan listing consumption in state a and state b.)

For the next question, assume the rate of interest on the bond (lending) is 0 percent and the consumer maximizes

$$U(C_1) + E_1 U(C_2) \, .$$

(These changes are just to simplify calculations.)

B) (15%) Now assume that a stock (equity) exists besides the safe bond. Let the amount of equity bought be S (it can be negative). Assume that the stock has a (net) rate of return of 0% if state a occurs [meaning that agent gets back the principal] and 100% if state b occurs. Find B and S and the implied consumption plan. (Note: the question is set up with "extreme" values to make the algebra easier, so the solution may also be "extreme." Also note, that for the PIH negative values of consumption are valid. If you are running out of time, most points will be accrued when you write down the equations that determines the answer.)

2. (15% of the January 2015 Core exam. No-one got parts B and C right, but I hope you will.) A consumer lives for 3 periods (periods 1, 2, and 3), earns $100 in the first period and the distribution of future earnings follows a uniform distribution on the interval [90,110] in periods 2 and 3. The consumer has a quadratic utility function and is—in period 1—allowed to freely borrow and lend at an interest rate that equals his or her rate of time preference which we for simplicity set to 0 (i.e., the net rate of interest is 0). The consumer is allowed to save in period 2 but not to borrow and the consumer has access to no other assets. Let C_i be the consumption of the representative consumer in period i.

A) Is $C_1 = E(C_2)$?

B) Is $E_1(C_2) = E_1(C_3)$? (Explain.)

C) Does the consumer save in period 1? (Argue why, and if savings are not zero, explain if they are positive or negative.)

3. (This may be hard, but see how far you get. It illustrates very clearly how future credit constraints may affect current consumption.) Two consumers live for 3 periods (periods 1, 2, and 3), earns $100 in the first period and the distribution of future earnings l 90 or 110 with probability 0.5 in each of periods 2 and 3. The consumers have a quadratic utility function and is—in period 1—allowed to freely borrow and lend at an interest rate that equals their rate of time preference which we for simplicity set to 0 (i.e., the net rate of interest is 0). Consumer A is allowed to save in period 2 but not to borrow and the consumer has access to no other assets. Consumer B can borrow and save. Let C_t^A, C_t^B be the consumption of consumer A and B, respectively, in period t.

A) Is $E_1(C_2^B) = E_1(C_3^B)$? (Explain.)

B) Find C_1^B.

Now assume that the agents get a further 10 dollars in period 3, and that this is known in period 1. (This is supposed to capture the logic of an announced tax break.)

C) Is $C_1^i = E(C_2^i)$, for $i = A, B$?

D) Find C_1^A and C_1^B under the new assumption. For which agent did consumption increase more (or, roughly, which agent has the larger propensity to consume).