ECONOMICS 7344, Spring 2019
Bent E. Sørensen

HOMEWORK 5. Due Wednesday April 24.

1. (36% of core exam June 2009) Assume an economy consists of two agents (home and foreign) who each maximize a von Neumann-Morgenstern utility function

$$U(C_0) + E_0 U(C_1) \ ,$$

where $U(C_t) = -\frac{1}{\gamma} \exp^{-\gamma C_t}$ where γ is a positive constant. Assume there is no storage and there are perfect Arrow-Debreu markets. There are two time periods ($t = 0$ and $t = 1$) and two states of the world, "A" and "B," in period 1. Assume there are N consumers in the economy.

a) Demonstrate (do the derivations) that this economy allows for a representative agent.
b) Derive a formula for the relation between the consumption of each agent and aggregate consumption.

Now assume that $N = 2$, $\gamma = 1$, and the endowment of the first agent is $y_0^1 = 3$, $y_1^1 = 5$ in state A and $y_1^1 = 1$ in state B. The endowment of the second agent in period 0 is $y_0^2 = 3$ and in period 1 his or her endowment is $y_1^2 = 1$ in state A and $y_1^2 = 5$ in state B. Assume that state A happens with probability 1/2.

c) For now assume that agents can trade in a bond but no other financial assets exist. Find the rate of interest.
d) Explain why the rate of interest is positive or negative using concepts from the class.
e) Now assume that the agents can trade in Arrow securities for state A and state B. Find the prices of the Arrow securities and the rate of interest. Explain why is it higher or lower than the rate of interest you found in question c).
f) Under the assumptions of question e), find the consumption of each agent in each period and in each state of the world.

2. (20% of midterm 2, 2009) Consider the CAPM-model. a) Assume the world only have two outcomes ("states of the world"). Let X be an asset whose payout PO_X is 100 if "shine" a situation where the (net) market return is 10 percent. "Shine" has probability 0.5. If "rain," PO_X is 200, the net market return is 0, "rain" also has probability 0.5. Assume that the safe rate of interest is 2 percent.

a) What is the expected return (ER_X) to an investment in X?

b) What are the possible returns R_X and their probabilities (in other words, what is the distribution of R_X).

3. (7% of second core exam 2003.) Asset A and asset B exist for one period and their returns have identical covariances with the market return. Assume that the market return is higher than the safe rate of interest (you should always make that assumption unless instructed otherwise). The rate return of asset B has a variance that is twice as large as the variance of the rate of return of asset A. Which asset will—if the CAPM holds—have the highest expected rate of return?

4. Assume that the mean return on the market portfolio (ER_M) is 10% and that a safe asset exists with a return of 4%. Assume that the standard CAPM is true.

a) Let X is an asset whose payout is determined by you flipping a coin and paying 1$ if head and nothing if tail. What is the return (R_X) to an investment in X?

b) Now let the return (R_i) to an asset be $R_i = .5\, R_M + .5\, R_X$. What is the expected value $E(R_i)$.

c) If the asset X now paid out 100$, rather than just 1$, in the case of heads, and still nothing in the case of tails. What would now be the answer to a)?

5. Assume that IBM stock has a mean return of 3% and a variance of 4, and that GM stock has a mean return of 10% and a variance of 9. Also assume that the covariance between IBM and GM stock is 1. Calculate the mean and standard deviation for portfolios that consist of IBM and GM stocks: do this for 0, 25%, 50%, 75%, and 100% invested in IBM. Sketch (by hand) the efficient frontier when these are the only assets available.