ECONOMICS 7344, Spring 2019

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HOMEWORK 4. Due Wednesday April 17.

1. (Partial equilibrium.) Consider an agent with income ("output" in Obstfeld-Rogoff) $Y_1 = 10$, $Y_2^A = 16$, and $Y_2^B = 6$, where A and B are states of the world with $\pi^A = 0.5$ and $\pi^B = 0.5$. Assume $p^A = p^B$, $r = 10\%$ and the discount rate is $\beta = \frac{1}{1+r}$.

a) Assume the agent has quadratic utility and that the agent can trade in Arrow-securities for both state A and state B. Does the "PIH-relation" $C_1 = EC_2$ hold?

b) Find C_2^A/C_2^B.

c) How many units of each Arrow-security does the agent purchase and how many units of the period 1 good? (this can be a negative number so "purchase" may mean sell.)

Now assume that the agent has utility function $U(C) = -\frac{1}{3}C^{-3}$.

d) Find C_2^A/C_2^B. (Give the intuition for why it does or does not change from the answer in part b). [This is probably a hard question]).

e) Find C_1.

f) Now assume $\frac{p^A}{p^B} = \frac{2}{3}$. Now find C_1 and C_2^S for $S = A, B$ and check if $C_1 = EC_2$.

2. (Use the note on market spanning.) Consider the case of an economy with four states-of-the-world. Assume that an asset S_1 exists that pays 2 units in period 1 if state A occurs, 1 unit if state B occurs, and nothing if state C or D occurs. Another asset S_2 exists which pays 1 unit in period 1, if state C occurs, and nothing in states A, B, and D. A third asset S_3 pays 0 units in period 1 if state C occurs, and 2 units in states A, B, and D. Finally, a

discount bond paying one unit in period 1 for sure can be traded.

a) Is the set of assets equivalent to a full set of Arrow securities?

b) Now assume that asset S_3 instead pays 1 unit in period 1, if state A occurs, and 0 units in states B, C, and D. Are the markets perfect (equivalent to a full set of Arrow securities) in this case?

3. (This is general equilibrium that we will cover Monday.) (15% of the June 2014 core exam) Consider the case of the 2 agents, 2 periods, 2 states-of-the-world model of Obstfeld-Rogoff Chapter 5.2 (where agents can trade using a full set of Arrow securities). Assume that both agents have utility functions $U(C_0) + E_0U(C_1)$, where $U(C_t) = -\exp(-C_t)$.
Assume that the endowment of the first agent is $y_0 = 3, y_1 = 3$ and that the endowment of the second agent in period 0 is $y_0^* = 3$ and in period 1 his or her endowment is $y_1^* = 6$ in the "good state" g. In the "bad state" b the endowment of the second agent is $y_1^* = 0$. Assume that the good state happens with probability 0.5.
a) Derive the formula for the rate of interest as a function of initial endowments and period 1 endowments.
b) Now assume that in period 1 the endowment of the second agent is $y_1^* = 4$ in the "good state" and $y_1^* = 2$ in the "bad state." Assume that the good state happens with probability 0.5. Will the rate of interest go up or down compared to the initial situation? (It is more important that you argue the logic than solving for numbers—you don't have to do numbers at all in the question if you argue clearly.)