ECONOMICS 7344, Spring 2018
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HOMEWORK 5. Due Wednesday April 18.

1. Consider the case where utility is exponential $U(C) = -\exp(-C)$ and C_t is distributed i.i.d. (independently, identically, distributed) $N(\mu, \sigma^2)$. The consumer maximises $\Sigma_{t=0}^{\infty} 0.9^t E_0 U(C_t)$.

a) What is the (period 0) price of a one-period discount bond?

b) What is the price of a two-period discount bond?

Now assume that utility is exponential $U(C) = -\frac{1}{4}\exp(-4C)$.

c) What is the price of the one period discount bond? Explain why it is now different.

Now assume that $C_t = \mu + \alpha C_{t-1} + u_t$, where u_t is i.i.d. $N(0, \sigma_U^2)$.

d) What is the price of a two-period discount bond?

e) What happens to the price of the bond if the variance of u_t doubles? What is the intuition for that?

2. Consider an agent with income ("output" in Obstfeld-Rogoff) $Y_1 = 10$, $Y_2^A = 18$, and $Y_2^B = 2$, where A and B are states of the world with $\pi^A = 0.5$ and $\pi^B = 0.5$. Assume $p^A = p^B$, $r = 10\%$ and the discount rate is $\beta = \frac{1}{1+r}$.

a) Assume the agent has quadratic utility and that the agent can trade in Arrow-securities for both state A and state B. Does the "PIH-relation" $C_1 = EC_2$ hold?

b) Find C_2^A / C_2^B.

c) How many units of each Arrow-security does the agent purchase and how many units of the period 1 good? (this can be a negative number so "purchase" may mean sell.)

Now assume that the agent has utility function $U(C) = -\frac{1}{3}C^{-3}$.

d) Find C_2^A / C_2^B. (Give the intuition for why it does or does not change from the answer in part b). [This is probably a hard question]).

e) Find C_1.

f) Now assume $\frac{p^A}{p^B} = \frac{2}{3}$. Now find C_1 and C_2^S for $S = A, B$ and check if $C_1 = EC_2$.

3. Consider the case of an economy with four states-of-the-world. Assume that an asset S_1 exists that pays 2 units in period 1 if state A occurs, 1 unit if state B occurs, and nothing if state C or D occurs. Another asset S_2 exists which pays 1 unit in period 1, if state C occurs, and nothing in states A, B, and D. A third asset S_3 pays 0 units in period 1 if state C occurs, and 2 units in states A, B, and D. Finally, a discount bond paying one unit in period 1 for sure can be traded.

a) Is the set of assets equivalent to a full set of Arrow securities?

b) Now assume that asset S_3 instead pays 1 unit in period 1, if state A occurs, and 0 units in states B, C, and D. Are the markets perfect (equivalent to a full set of Arrow securities) in this case?