

HOMEWORK 4. Due Wednesday April 11.

1. (20% of midterm 2, 2009) Consider the CAPM-model. a) Assume the world only have two outcomes (“states of the world”). Let X be an asset whose payout PO_X is 100 if “shine” a situation where the (net) market return is 10 percent. “Shine” has probability 0.5. If “rain,” PO_X is 200, the net market return is 0, “rain” also has probability 0.5. Assume that the safe rate of interest is 2 percent.

a) What is the expected return (ER_X) to an investment in X ?

b) What are the possible returns R_X and their probabilities (in other words, what is the distribution of R_X).

2. Assume that an economy is inhabited by identical agents (or a representative agent) with logarithmic utility functions.

a) If the (non-stochastic) growth rate of consumption is 6 percent from period 1 to period 2 and agents have a discount factor of 0.99 (you can approximate the discount rate by 0.01), what is the safe rate of interest (the rate of interest from period 1 to period 2)?

b) Maintain the assumptions of part (a) but now assume the expected growth rate of consumption from period 2 to period 3 is 8 percent. Using the Euler Equation for the representative agent, find the (period 1) forward rate of interest for period 2 to period 3. (For this question only, you can make the approximation that $E\frac{1}{X} = \frac{1}{EX}$ for a certain variable X . This approximation is NOT allowed unless it is explicitly said so.)

3. (7% of second core exam 2003.) Asset A and asset B exist for one period and their returns have identical covariances with the market return. Assume that the market return is higher than the safe rate of interest (you should always make that assumption unless instructed otherwise). The rate return of asset B has a variance that is twice as large as the variance of the rate of return of asset A. Which asset will—if the CAPM holds—have the highest expected rate of return?

4. Assume that the mean return on the market portfolio (ER_M) is 10% and that a safe asset exists with a return of 4%. Assume that the standard CAPM is true.

a) Let X is an asset whose payout is determined by you flipping a coin and paying 1\$ if head and nothing if tail. What is the return (R_X) to an investment in X ?

b) Now let the return (R_i) to an asset be $R_i = .5 R_M + .5 R_X$. What is the expected value $E(R_i)$.

c) If the asset X now paid out 100\$, rather than just 1\$, in the case of heads, and still nothing in the case of tails. What would now be the answer to a)?

5. Assume that IBM stock has a mean return of 3% and a variance of 4, and that GM stock has a mean return of 10% and a variance of 9. Also assume that the covariance between IBM and GM stock is 1. Calculate the mean and standard deviation for portfolios that consist of IBM and GM stocks: do this for 0, 25%, 50%, 75%, and 100% invested in IBM. Sketch (by hand) the efficient frontier when these are the only assets available.