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ECONOMICS 7344 – MACROECONOMIC THEORY II, part b, Spring 2018

Homework 1. March 7, (may be updated with another question on March 14), due Wednesday March 21.

1. (From the January 2011, make-up core exam.) A consumer lives for 2 periods and earns $Y_1 = 10\$$, in period 1, and in period 2 he or she earns $Y_2^a = 10\$$ with probability 1/2 (state a) and $Y_2^b = 30\$$ with probability 1/2 (state b). The consumer starts with 0 assets and maximizes

$$U(C_1) + \frac{1}{1.10} E_1 U(C_2) ,$$

where

$$U(C) = 100C - \frac{1}{2}C^2 .$$

Assume that the safe rate of interest is 10 percent.

A) (5%) Let B denote the amount lent in period 1 (or, equivalently, the amount of a safe bond bought). Assuming that the agent only have access to a safe bond, find B and consumption in each period (for period 2, that means the consumption plan listing consumption in state a and state b.)

For the next question, assume the rate of interest on the bond (lending) is 0 percent and the consumer maximizes

$$U(C_1) + E_1 U(C_2) .$$

(These changes are just to simplify calculations.)

B) (15%) Now assume that a stock (equity) exists besides the safe bond. Let the amount of equity bought be S (it can be negative). Assume that the stock has a (net) rate of return of 0% if state a occurs [meaning that agent gets back the principal] and 100% if state b occurs. Find B and S and the implied consumption plan. (Note: the question is set up with "extreme" values to make the algebra easier, so the solution may also be "extreme." Also note, that for the PIH negative values of consumption are valid.

If you are running out of time, most points will be accrued when you write down the equations that determines the answer.)

2. (15% of the January 2015 Core exam. No-one got parts B and C right, but I hope you will.) A consumer lives for 3 periods (periods 1, 2, and 3), earns \$100 in the first period and the distribution of future earnings follows a uniform distribution on the interval [90,110] in periods 2 and 3. The consumer has a quadratic utility function and is—in period 1—allowed to freely borrow and lend at an interest rate that equals his or her rate of time preference which we for simplicity set to 0 (i.e., the net rate of interest is 0). The consumer is allowed to save in period 2 but not to borrow and the consumer has access to no other assets. Let C_i be the consumption of the representative consumer in period i.
A) Is $C_1 = E(C_2)$?
B) Is $E_0(C_2) = E_0(C_3)$? (Explain.)
C) Does the consumer save in period 1? (Argue why, and if savings are not zero, explain if they are positive or negative.)

3. Define the lag polynomials $a(L) = a_0 + a_1 L$ and $b(L) = b_0 + b_1 L + b_2 L^2$. (Notice: in the notes, and in class, it is often assumed $a_0 = 1$ and $b_0 = 1$. This is just for simplification and doesn't matter for any results since you can always re-scale the data and the lag-polynomial such that the first coefficient becomes unity (write a(L) as $a_0\, a'(L)$ where the lag polynomial $a'(L) = 1 + \frac{a_1}{a_0} L$ and similarly for $b(L)$). The constant a_0 will not affect the properties of the lag-polynomial that we care about.)

Assume $a_0 = 2$, $a_1 = -2$, $b_0 = 1$, $b_1 = -.3$, and $b_2 = .5$.
i) If $x_{t-1} = 2, x_{t-2} = -3, x_{t-3} = -2$, and $x_{t-4} = 9$, what is $a(L)x_t$? and $b(L)x_t$?
ii) Find the roots of $a(L)$ and $b(L)$.
iii) What is $c(L) = a(L)b(L)$?
iv) Find the roots of $c(L)$. Is the AR-model $c(L)x_t = 8 + u_t$ stable?
v) Find the coefficients to the constant (identify), L, and L^2 in the lag-polynomial $b^{-1}(L)$.
Now define the polynomials $d(x) = 1 + .2\, x$ and $e(x) = 1 + .5x^2$.
vi) Find the roots of d(x) and e(x). Are d(L) and e(L) invertible?

4. (24% of midterm 1, Spring 2005) Assume that income follows the AR(1) process

$$y_t = 2 + 0.4 y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance 3.

a) Is this time-series process stable?
b) Assume that y_0 is a random variable. For what values of the mean $E(y_0)$ and the variance $\mathrm{var}(y_0)$ will the time series y_t; $t = 0, 1, 2, ...$ be stationary?
c) What is $E_1 y_3$ if $y_1 = 5$ and $y_0 = 2$?
d) Write the infinite Moving Average model that is equivalent to the AR(1) model (*) [assuming that the process now is defined for any integer value of t]. (Half the points are from getting the correct mean term.)