ECONOMETRICS II, Fall 2020.

Bent E. Sørensen

Notes on Maximum Likelihood.

Consider a sample of *dependent* variables $x_1, ..., x_T$ with density $f(x_1, ..., x_T; \theta)$, where θ is a $k-$vector of parameters. We will mainly consider the multivariate Normal with mean μ and variance Σ. To estimate μ and variance Σ you find

$$\widehat{\mu, \Sigma} = \text{argmax}_{\mu,\Sigma} \frac{1}{((2\pi)^N |\Sigma|)^{0.5}} \exp\{-0.5(X-\mu)'\Sigma^{-1}(X-\mu)\}.$$

or, equivalently,

$$\text{argmax}_{\mu,\Sigma} - \log(|\Sigma|) - (X-\mu)'\Sigma^{-1}(X-\mu)\,.$$

This is of course impossible because of the dimension of Σ so we assume that the data follows, say, an ARMA model, which makes Σ a function of a low number of parameters. We will illustrate in a homework that you directly minimize the multivariate likelihood function in cases where you cannot find a formula for Σ^{-1}. (This is case for the MA models.) In most cases, we make use of conditioning. Recall that $f(x|y)$ is $f(x,y)/f(y)$ or $f(x,y)$ is $f(x|y)\,f(y)$ so

$$f(x_1, ..., x_T; \theta) = f(x_T | x_1, ..., x_{T-1}; \theta) f(x_1, ..., x_{T-1}; \theta)$$

But you can do the same for $f(x_1, ..., x_{T-1}; \theta)$ and on and on, till you have

$$f(x_1, ..., x_T; \theta) = f(x_T | x_1, ..., x_{T-1}; \theta) f(x_{T-1} | x_1, ..., x_{T-2}; \theta) .... f(x_2 | x_1; \theta) f(x_1; \theta) \, .$$

We usually write $f(x_t | x_1, ..., x_{t-1}$ as $f(x_t | X^{t-1})$ or sometimes more compactly as $f_{t-1}(x_t; \theta)$.

You can think of AR models as ways to write probabilities conditional on the past in a simple way (we will elaborate in the next handout).

We can take logs as before and get the log likelihood function

$$log f(x_1; \theta) + log f_1(x_2; \theta) + ... + log f_{T-1}(x_T; \theta) \, .$$

For stationary models, the LLN and CLT we used in the ML theory still holds (not for a random walk). So the powerful results regarding the asymptotic variance and the Hessian etc. still holds.