MACROECONOMETRICS, Spring 2022

Homework 2. Due Wednesday February 9. Remember to comment on what you see. I will call on some of you to share your plots with the class.

1. (Unit root distribution) Simulate and estimate the model

$$y_t = \rho y_{t-1} + e_t \ ,$$

where $y_0 = 0$ and the e_t innovations are standard normals, independent across t. Set $T = 50$ and 500.

Do, say, 50 simulations for each value of T and for values of $\rho = 0.1, 0.95, 0.99$ and 1.0 and plot the distribution of $\hat{\rho}$ and their empirical t-statistic. (e.g., make histograms or whatever, as long as the distribution can be seen).

2. (Dickey-Fuller test. Distribution under null of unit root) Simulate the model

$$y_t = \mu + \rho y_{t-1} + e_t \ ,$$

where y_0 is fixed and e_t are standard normals, independent across t. Set $\mu = 0$ and let $T = 50$ and 500.

Run the regression

$$\Delta y_t = \gamma_0 + \gamma_1 \, t + \gamma_2 y_{t-1} + \gamma_3 \Delta y_{t-1} + u_t \, . \quad (**)$$

(Note here, that if $\rho < 1$, if you difference each side of (*), you get (**) with $\gamma_2 = 0$, and $\gamma_3 = \rho$. We don't worry about the intercept, but usually always include one, except if there are strong reasons [from economics] not to and we usually allow for a trend. But if $\rho = 1$, then Δy_t is white noise around a trend with slope μ and γ_2 is zero, and non-stationary so it would not make any sense if γ_2 were not 0. γ_1 also has to be zero to avoid quadatic trend.)

Do, say, 50 simulations for each value of T and for $\rho = 1.0$ and plot the distribution of $\hat{\gamma}_2$ and the empirical t-statistics.

3. (Dickey-Fuller test. Power) Simulate the model

$$y_t = \mu + \delta \, t \ + \ \rho y_{t-1} + e_t \ ,$$

where $y_0 = 0$ and e_t are standard normals, independent t. Set $\mu = 1$ and $\delta = 0.5$. Let $T = 50$ and 500.

Run the regression

$$\Delta y_t = \gamma_0 + \gamma_1 \, t + \gamma_2 y_{t-1} + \gamma_3 \Delta y_{t-1} + u_t \, . \quad (**)$$

Do, say, 50 simulations for each value of T and for $\rho = .95$ and plot the distribution of $\hat{\gamma}_2$ and the empirical t-statistics. (The "Augmented" Dickey-Fuller test that you might use in practice, will typically have some more lagged Δy_{t-k} terms. If you want to play more, try and include some more lags in (**) and see how it changes the critical value for a one-sided 5% test. You would want as many lags as you think are in the true process in order to not be mis-specified and then the critical values changes a bit.)

4. (Spurious regressions) Simulate and estimate the model

$$y_t = \rho y_{t-1} + e_t \ ,$$

and, using independently drawn errors

$$x_t = \rho xt - 1 + u_t \ ,$$

where y_0 and x_0 are fixed at 0 and e_t and u_t are standard normals, independent of each other and across t. Set $T = 50$ and 500.

Do, say, 50 simulations for each value of T and for values of $\rho = 0.95$ and 1.0 and regress y on x; i.e., run the regression

$$y_t = \mu + \alpha xt + v_t \ ,$$

Plot the distribution of $\hat{\alpha}$ and their empirical t-statistics. Also show the mean and median. Comment.