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The effect of interest rates on consumption

Consider the Euler Equation and let us for the moment assume that the interest rate is time-varying and known (i.e., the interest rate is non-stochastic). The Euler Eq. is then

$$u'(c_t) = \frac{1 + r_{t+1}}{1 + \rho} E_t u'(c_{t+1}) \ .$$

For the special case of a CRRA utility function $u(c) = \frac{c^{1-\theta}}{1-\theta}$ (where θ is positive) this implies

$$c_t^{-\theta} = (\frac{1 + r_{t+1}}{1 + \rho}) E_t c_{t+1}^{-\theta}$$

or (using the [innocuous] approximation $\log(1 + x) = x$ and the [more serious] approximation $\log E_t c_{t+1}^{-\theta} = E_t \log c_{t+1}^{-\theta}$

$$E_t \Delta \log c_{t+1} = \frac{1}{\theta}(r_{t+1} - \rho) \ .$$

This equation has some clear implications since θ is positive. **The higher the interest rate the higher the expected growth rate of consumption**. This conclusion is likely to hold in more complicated models with uncertainty. (Note that for no uncertainty or quadratic utility we don't need to make the "more serious approximation." In general, be careful about using the approximation $\log E(X) = E \log(X)$, usually it will cost you points. We use it here since this allows us to show a point very simply that holds more generally.)

Note, however, that it is not obvious if consumption in period t "increase" or "decrease" in response to an unexpected change in the interest rate r_{t+1}. Be careful here: By "increase" I mean if $\frac{\partial c_t}{\partial r_{t+1}}$ is positive, which is different from the question of whether the growth rate in consumption from t to $t + 1$ increase. An increase in the interest rate r_{t+1} may increase or decrease c_t, this depends on the income vs. the substitution effect. We are, however, fairly confident from the above the the interest rate increase will increase the growth rate of consumption from period t to $t + 1$. However, Federal Reserve policy makers are often more interested in the effect on the *level* of consumption.

To illustrate the income versus the substitution effect, I will go over Figure 8.2 on p. 382 in Romer, 4th edition.