Macro II, Spring 2023

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Efficient Markets.

In many financial textbooks the Efficient Market Hypothesis (EMH) is treated prominently (see Chapter 2 of the Campbell, Lo, MacKinlay text or any undergraduate finance book on investment). "Efficient" financial markets should probably price assets correctly—reflecting expected future earnings and allowing for a reasonable reward for accepting uncertain returns ("risk premia"). Campbell, Lo, MacKinlay consider the issue of prices verses future dividends; but this is an area that is much harder than one should think, mainly because it is hard to know what are the discount rates that agents may use for future earnings.

Usually "EMH" refers to the simpler question of whether asset prices are *predictable*. The simple logic is that if agents could predict that an asset would increase in value, this could not be an equilibrium (everyone would buy, increasing the price instantly until it would equal the expected future price). Starting with Eugene Fama of the University of Chicago, financial economists no abnormal profits can be earned from trading in past patterns of returns to a stock. Fama became very famous (also earned a Nobel), so we start here and move to the modern Euler equation shortly.

Example 1: One might think that a stock that does well today will also do well (or maybe less well) tomorrow - this would happen if positive information that become available about a firm only gradually affects the market price. How do we test this? We run an autoregression of the form

$$r_{it} = \alpha_i + \beta_1 r_{it-1} + ... + \beta_l r_{it-l} + u_{it} \ .$$

If the EMH is true the $\beta_l = ... = \beta_1 = 0$, which you can test with an F-test. Most often you will include only one or two lagged variables in the regression In other words, the weak form EMH states that the return of any stock (or portfolio) should be unpredictable "white noise."
If the return series is white noise this means that the expected value of the stock *price* tomorrow is equal to the stock price today (when we correct for the safe return). This is the definition of a martingale but people tend to say random walk (which is fine, so I will also do it, just don't do it

in a job interview). The random walk model takes the form

$$p_{it} \;=\; \alpha_i + p_{it-1} + u_{it} \;, \qquad (*)$$

where p_{it} is the price of asset i at time t (corrected for dividends). The mean α_i captures the safe rate of interest and a risk premium.[1] Often you will see the random walk hypothesis stated as

$$p_{it} \;=\; p_{it-1} + u_{it} \;,$$

(i.e., without any drift term), which is reasonable for daily data because the daily safe rate of interest (and the risk premium) is very small. One can perform tests for EMH directly on the stock prices rather than looking at returns, but this is complicated by "unit-root" econometric issues so we often prefer to look at returns.

If prices are random walks, then the expected value of today's price is yesterday's price, i.e. $E\{p_{it}\} = p_{it-1}$, which approximately implies that $E\{\ln p_{it} - \ln p_{it-1}\} = 0$. Since

$$E\left\{\ln p_{it} - \ln p_{it-1}\right\} = E\left\{\ln\left(p_{it}/p_{it-1}\right)\right\} = E\left\{\ln\left(1 + \frac{p_{it} - p_{it-1}}{p_{it-1}}\right)\right\} = E\left\{\ln\left(1 + r_{it}\right)\right\} \approx E\, r_{it} \;,$$

it follows that prices being random walks is equivalent to expected returns being 0. (More precisely, this should be all be done in terms of excess returns. Also note that we ignore dividends here, on a dividend day, the price should be corrected by setting p_t equal to the new lower price and adding the dividend.) Typically, one estimates autoregressive models and tests if the coefficients to the lagged variables are zero.

Pricing a payoff with the Euler Equation

If we consider the payoff to an asset that you can buy today and which pays off tomorrow then we can find the price using the Euler equation.

Consider an asset i with a payoff tomorrow of PO_i (I suppress the time index, in many applications, tomorrow's payoff is the value of the asset at $t+1$). The net return to a dollar investment

[1] α_i is called the *drift* since you can solve the equation "recursively." Note that the equation holds for all t so by recursive substitution you get $p_{it} \;=\; t\alpha_i + p_{i0} + u_{i1} + u_{i2} + \ldots + u_{it}$, so the expected value of the price goes up as a line with slope α_i.

in asset i is now

$$r_i = \frac{PO_i}{P_i} - 1 \qquad (a)\ ,$$

where P_i is today's price. Assume that the correlation of PO_i with tomorrow's marginal utility $U'(c_{t+1}$ is known. Then the Euler equation becomes

$$U'(c_t) \;=\; \beta E\{U'(c_{t+1})\frac{PO_i}{P_i}\}\ ,$$

where $\beta = 1/(1+\rho)$ is the discount factor. This determines the price:

$$P_i \;=\; E\{\beta \frac{U'(c_{t+1})}{U'(c_t)} PO_i\}\ ,$$

or, if we define,

$$m_t = \beta \frac{U'(c_{t+1})}{U'(c_t)}$$

we have

$$(*) \quad P_i \;=\; E\{m_t PO_i\}$$

for any asset i. As John Cochrane points out in his graduate book on asset pricing, most asset pricing models have the form (*) for different definitions of m_t which is often called a "pricing kernel" in more abstract treatments in finance. (If you are particularly interested in asset pricing, you may want to check out Cochrane's web-page.) m_t has an interpretation as the discounted marginal value (utility) of returns.

The 2-by-2 case. We will stress the simplest case with 2 periods and 2 outcomes in period 2, in order to build up understanding (I will likely ask several "2-by-2 questions" on the exam).

Pricing a pay-off using the Euler equation (and assuming exogenous consumption) in the 2 period, 2 states-of-the-world case.

A consumer lives for 2 periods and consumes C_1, in period 1, and in period 2 he or she consumes C_2^a in state a and C_2^b in state b.

You want to find the price P_i of an asset that has a payoff in period 2 of PO_i^a in state a and PO_i^b

in state b. Assume that the conditional probability of state a and the conditional probability of state b is Pr^b (we often leave the word "conditional" out, but in this case it is implicit that it is conditional probabilities). As always use the Euler equation

$$U'(C_1) \;=\; \beta E\{U'(C_2)(1+r_{t+1})\} \;,$$

which becomes

$$U'(C_1) \;=\; \beta\,[Pr^a U'(C_2^a)\frac{PO_i^a}{P_i} + Pr^b U'(C_2^b)\frac{PO_i^b}{P_i}\,] \;.$$

Now solve for P_i:

$$P_i \;=\; \beta\;[\,Pr^a\,\frac{U'(C_2^a)}{U'(C_1)}\,PO_i^a + Pr^b\,\frac{U'(C_2^b)}{U'(C_1)}\,PO_i^b\,] \;.$$

If you fully understand that equation, you basically understand the full Lucas asset pricing model (with more periods, more agents, more states-of-the-world, the math may look more complicated, but the intuition will not change).

Observe:

- The price is proportional to the discount factor. The price is how much of period 1 good to hand over for the right to a random period 2 good, so the less you value the future the less you want to pay).

- The price is proportional to the payout in period 2 (meaning that if you, say, double *both* PO_i^a and PO_i^b, the price of the asset will double. This is obvious: you pay twice as much for twice as much.

- If an asset only has a payout in state a then the price is proportional to the probability that state a will happen (of course, same for b). An example of an asset that pay out in one state of the world is a lottery ticket. If you buy two tickets, you double the probability of getting the payout and, of course, you pay twice as much.

The price of the asset is also determined by relative scarcity. Because we assume that people's choices are described by strictly concave utility functions, agents will always want to use assets to transfer goods to the period and/or the state-of-the-world where consumption is lowest (of course still taking the previous points into accounts). [A note on the word "want:" The Lucas asset pricing model is about equilibrium prices, you cannot get the price that you want when buying anything. However, prices reflect demand and therefore people's "desires."] Regarding relative scarcity, we can observe:

- If consumption is relatively low in period 1 prices a lower. The intuition is that we prefer to consume more in period 1 relative to period 2 in this situation. The math is that the marginal utility $U'(C_1)$ is high which makes the right-hand side of the equation small.

- If consumption is relatively low in state a compared to state b. You will pay more for an asset that has a relatively high payoff in state a. The intuition is again scarcity and the math is that the payoff to state a are weighed by $U'(C_2^a)$ which is large if C_2^a is low.

Let us denote the real gross (net) return to the safe asset by R_t^f (r_t^f) where we sometimes drop the t subscript indicating that the safe interest is not time-varying. In the real world the interest will typically vary over time but the real interest rate is a lot less variable than, say, stock market returns and we therefore often assume it constant for notational simplicity. The safe asset satisfies the Euler equation, which in the "kernel notation" becomes:

$$1 = E_t\{m_{t+1}R_{i,t+1}\} \quad in \;\; particular \quad 1 = E_t\{m_{t+1}\}R^f \; .$$

(The price of an asset with payout equal to the gross return is 1), so the implication is that for any asset i,

$$E_t\{m_{t+1}(R_i - R^f)\} \;=\; 0 \; .$$

So theoretical economists agrees that excess returns should have expectation 0, but *only* after adjusting the return with the "stochastic discount factor" m_t. At short horizons m_t can typically be assumed to equal 1, so in this sense the Euler Equation agrees with the EMH. But even at short horizons an asset with a very high correlation with m_t should not have $E_t\{(R_i - R^f)\} \;=\; 0$. In order to calculate correlations, one obviously need to have specific model for m_t.

Note that because the safe rate of interest satisfies

$$R^f \;=\; 1/E_t\{m_t\}$$

and using that $E\{XY\} = EX\,EY + Cov(X,Y)$ we have that

$$0 \;=\; Em_{t+1}E\{R_i - R^f\} + Cov(R_i m_{t+1})$$

which implies that

$$E\{r_i - r^f\} = -R^f\,Cov(r_i m_{t+1})\ .$$

So the existence of a stochastic discount factor implies that excess returns are a function of the covariance of returns with the kernel. Several important models of asset pricing centers around such a relation.