ECONOMETRICS 1, Spring 2018
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Midterm 1- February 14, 2018

Each sub-question in the following carries equal weight.

1. (30%) Consider the model

$$Y \;=\; X\beta \;+\; \epsilon \;,$$

where X is an $N \times K$ matrix and Y and ϵ are N-vectors. Assume the standard assumptions for OLS holds.

a) What are the standard assumptions?
b) Find (derive) the OLS estimator $\hat{\beta}$.
c) Show that $X\hat{\beta}$ takes the form PY where P is an $N \times N$ symmetric idempotent matrix. (You have to show that it is symmetric and idempotent).
d) Show that $M = (I - P)$ satisfies $e = MY$, where e is the vector of residuals and verify that e is orthogonal to X.
e) Verify that the mean $\bar{e} = 0$ if one column in X is a vector ι of ones.

2. (20%) For the OLS estimator, prove the Frisch-Waugh theorem. (If you find it simpler, you are allowed to assume that there are only two regressors and solve the normal equations.)

3. (15%) Explain what is the Chow test and show how the test can be written using a formula that involved the sums of squares from three separate regressions.

4. (15%) Assume that you are interested in estimating the model (where you can treat the variables as having mean 0)

$$Y_i \;=\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \epsilon_i$$

by OLS. Assume that you have 80 observations and that you know the moments

$$X_1'X_1 = 3, \quad X_2'X_2 = 2, \quad X_1'X_2 = 0, \quad X_1'Y = 5, \quad X_2'Y = 4, \quad Y'Y = 20 \;.$$

a) Find the estimated coefficients b_1 and b_2.
b) Find the estimated variance $\hat{\sigma}^2$.
c) Perform a 5% two-sided t-test for the hypothesis $\beta_1 = 1$. (If you could not find s^2 in b) use a value of 2.0).

5. Computer question (20%). Read the Matlab code below and answer the questions in the code.

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Econometrics 1
% Spring 2018
% Midterm 1
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;
clc;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% This code estimates the model
%
%                    y = beta0 + beta1*X1 + beta2*X2 + e
%
% using OLS and calculates other things.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Generate the data.

n = 500;                          % Sample size

X1 = randn(n,1);                  % X1
X2 = randn(n,1);                  % X2

X = [ones(n,1) X1 X2];            % X matrix with constant

beta = [1; 3; 2];                 % True coefficients

u = randn(n,1);                   % Standard normal disturbances

y = X*beta + u;                   % Observed values of y

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% Estimate the coefficents using OLS.

b = inv(X'*X)*X'*y;             % OLS estimates

% Compute the standard errors.

k = size(beta, 1);              % Number of coefficients

yhat = X*b;                     % Predicted values of Y
uhat = y - yhat;                % Residuals

s2 = XXXX1;        % S Squared

vc = XXXX2;              % Variance-Covariance Matrix

se = XXXX3   % Standard Errors

% Compute the t-statistics.

t = XXXX4;                      % t-statistics

disp(' ')
disp('Model: y = beta0 + beta1*X1 + beta2*X2 + e')
disp(' ')

disp('Regression Results')
disp(' ')

disp('   Estimates    SE     |t-stat|')
disp([b se t])
disp('Note: OLS estimates are b0, b1 and b2 in that order.')
disp(' ')

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Question 1: Complete the code above  by replacing XXXX1-XXXX4 with the relevant Matlab code
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

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