Midterm Exam II—October 28, 2019

Each sub-question in the following carries equal weight except when otherwise noted.

1. (15%) Verify that

$$(A \otimes B)' = A' \otimes B'$$

for two by two matrices.

2. (20%)
Consider a stationary first-order VAR

$$Y_t = \mu + AY_{t-1} + u_t \,,$$

where $\mu' = \{2, 3\}$ and for Ω the variance-covariance matrix for the error terms: $\text{var}(u_t)$, we have

$$\Omega = \begin{pmatrix} 4 & 1 \\ 1 & 8 \end{pmatrix} ,$$

and

$$A = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} .$$

a) What is Ey_t. (A vector.)

b) Write down the formula for the variance of y_t. (Hint: You need to use a Kronecker product.)

c) What is $\text{Var}(y_t)$? (Here I want the numbers. You will get partial points for the right formulas and whatever you do correctly.)

3. (25%) a) In the following code, what is the object B_xxx calculated where there is an A:?

b) Is B_xxx a consistent estimator? (Explain.)

c) In the following code, what is the object B_yyy calculated where there is an B:?

```
    X = [ones(T,1) x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13];
for s = 1:sim

    u1 = normrnd(0,sigma1,T,1);                                       .
    u2 = normrnd(0,sigma2,T,1) + 5*u1;                              .

    y2 = beta3 + beta4*x1 + beta5*x2 + beta6*x3 + beta7*x4  + u2;
    y1 = beta0 + beta1*y2 + beta2*x1 + u1;

    Y = [y1 y2];
    Y1 = Y(:,1);   %same as y1
    Y2 = Y(:,2);   %endogenous regressors, same as generated y2


    X_exo1 = X(:,1:2); %exogenous regressors in the reduced form
    X_OLS=[ones(T,1) Y2 x1];

       B2_hat = inv(X'*X)*X'*Y2;
       Y2_hat = X*B2_hat;
        X1_hat = [ones(T,1) Y2_hat x1];
A:     B_xxx(s,:) = inv(X1_hat'*X1_hat)*X1_hat'*Y1;

   N = length(Y2);
    Mexo = eye(N) - X*inv(X'*X)*X';
        Mexo1 = eye(N) - X_exo1*inv(X_exo1'*X_exo1)*X_exo1';
    W = [Y1 Y2]'*(Mexo)*[Y1 Y2];
    W1 = [Y1 Y2]'*(Mexo1)*[Y1 Y2];   %2x2 matrix
    lambda = min( eig(inv(W)*W1 )) ;
B:    B_yyy(s,:) = inv(X_OLS'*(eye(N)-(lambda*Mexo))*X_OLS)*(X_OLS'*(eye(N)-(lambda*Mexo))*Y1)
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4. (20%) Consider the model :

$$z_i = b\,w_i + c\,x_i + e_i,$$

and

$$w_i = d\,z_i + f\,v_i + g\,x_i + u_i\,,$$

where z and w are random variables, x and v are exogenous random variables, b, c, d, f, and g are unknown parameters, and e and u are independent white noise terms.

a) Explain which (if any) of these equations one can estimate without bias.

b) Suggest a typical consistent estimator for the equation(s) that you can estimate. (You do not need to write down any formula.)

5. (20%) Consider a simple OLS estimation

$$y_t = \mu + \alpha x_t + e_t \,,$$

where the error term is white noise.

a) Explain how you would construct a test for significance of α using a parametric bootstrap. (This is not something we likely would do for a linear model, but that is not the point here.)

b) Explain how you would test for significance of α using the non-parametric bootstrap that we covered in class.