

Midterm 1—September 25, 2025.

Each sub-question in the following carries equal weight except if otherwise noted.

1. (20%) Consider the AR model

$$y_t = \mu + a y_{t-2} + u_t,$$

where the error term is white noise with variance σ_u^2 .

- a) For which values of a is this model stable?
- b) Find the mean of y_t in the case where the model is stationary.
- c) Find the variance of y_t in the case where the model is stationary.
- d) Find the first order covariance (still assuming stationarity).

2. (20%) Consider the model

$$y_t = \mu + u_t,$$

where

$$u_t = \alpha u_{t-1} + v_t$$

is stationary and v_t is i.i.d. $N(0, \sigma^2)$.

- a) Write down the likelihood function for a sample of N observations.
- b) Mention 2 ways (at least) we can estimate μ (these would be models that are not ML).

3. (20%) Assume you have estimated 3 parameters, with estimates $\hat{\beta}_1 = 7$, $\hat{\beta}_2 = 9$, and

$\hat{\beta}_3 = 11$, and assume that you know for sure the estimates are normally distributed and the variance-covariance matrix is

$$\Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 1 & 4 \end{pmatrix} \quad \text{and} \quad \Sigma^{-1} = \frac{1}{11} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & -1 & 3 \end{pmatrix}.$$

- a) Write down the Wald test statistic for $\beta_2 + \beta_3 = 2$. (I want you to write this as a scalar, a real number, although you do not have to solve for ratios or invert matrices in the questions here.)
- b) Write down the Wald test statistic for $(\beta_2, \beta_3) = (2, 1)$. (I want you to write this as a scalar also.)
- c) Write down a the Wald test statistic for the hypothesis $\log(\beta_1) - \beta_3^2 = 2$.
- d) What is the distribution of the test statistic you wrote down? (Is this an asymptotic test or an exact test?)

4) (10%) Write down the probit model for 3 ordered outcomes.

5. (15%) Matlab question.

- a) Explain what the code does (what model is being estimated?).
- b) Fill in the missing line in the code. (It does not have to be perfect Matlab code.)

```
function [ L ] = logl_xx( b0 )
```

```
global x T
```

```
mean = b0(1);           % Mean.
stddev = b0(2);         % Standard deviation.
theta1 = b0(3);         % .
```

```
L = FILL IN THE CODE HERE
```

```
for t = 2:T
    L = L - 0.5*log(2*pi) - 0.5*log(stddev^2) ...
        - 0.5*(((x(t) - mean - theta1*x(t-1))^2/stddev^2));
end

L = -L;                % Negative of logl(for minimization).

end
```

6. (15%) Matlab question.

a) Explain what the code is supposed to do.

b) Explain intuitively what happens if $\rho=0$. (Which model(s) is (are) estimated in this case?)

```
function [L] = logl_ss(b)
```

```
global x w y z N
```

```
b0 = b(1);
b1 = b(2);
sigmau = b(3);
g0 = b(4);
g1 = b(5);
rho = b(6);
```

```

XB = b0*ones(size(x,1),1) + b1*x ;
WG = g0*ones(size(w,1),1) + g1*w ;

L=0 ;

for i = 1:N
    if z(i) == 1
        L = L + log((1/sigmau)*normpdf((y(i) - XB(i))/sigmau))...
            + log(normcdf((1/sqrt(abs(1-rho^2)))*(WG(i) + (rho/sigmau)*(y(i) - XB(i)))))
    elseif z(i) == 0
        L = L + log(normcdf(-WG(i)));
    end
end

L = -L;

end

```