ECONOMETRICS II, Fall, 2019. 5 questions.
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Midterm Exam 1—Monday, September 30th, 2019

Each sub-question in the following carries equal weight except when otherwise noted.

1. (20%)
Consider the AR(2) model

$$y_t = \mu + a\, y_{t-1} + b\, y_{t-2} + u_t \,,$$

where the error term is white noise. Assume the model is stationary.

a) Find (derive) the mean of y.

b) Find the first order autocovariance. (It is a complete answer with the variance in the solution. You will find the variance in the next question.)

c) Find the variance.

d) Find the variance of y_t conditional on y_{t-1}. (Note that you only condition on y_{t-1} not on previous values.)

2. (30%) Below are two pieces of Matlab code. Explain what these likelihood functions estimate. (You will get points for the correct names, but for full points you should briefly explain the logic of what is being done.)

A. First Code:

```
function [L] = logl_ss(b)

global x w y z N

b0 = b(1);
b1 = b(2);
sigmau = b(3);
g0 = b(4);
```

```
XB = b0*ones(size(x,1),1) + b1*x ;
WG = g0*ones(size(w,1),1) + w ;

L=0 ;

for i = 1:N
  if z(i) == 1
    L = L + log((1/sigmau)*normpdf((y(i) - XB(i))/sigmau))...
            + log(normcdf(WG(i)));
  elseif z(i) == 0
    L = L + log(normcdf(-WG(i)));
  end
end

L = -L;
end
```

B. Second code:

```
global x z N

b0 = b(1);
b1 = b(2);
b2 = b(3);

XB = b0*x ;

L=0 ;
for i = 1:N
  if z(i) == 0
    L = L + log(normcdf(b1 - XB(i)));
  elseif z(i) == 1
    L = L + log(normcdf(b2 - XB(i)) - normcdf(b1 - XB(i)));
  elseif z(i) == 2
    L = L + log(normcdf(XB(i) - b2));
  end
end

L = -L;

end
```

3. (20%) Consider a model

$$y_i = \mu + ax_i + u_i \,,$$

where u_i is normally distributed with (mean 0) and variance σ^2. Assume that y_i is only observed if

$$z_i = 1 \,,$$

where

$$z_i = 1 \;\; \text{if} \quad z_i^0 > 0 \,,$$

where

$$z_i^0 = w_i \gamma + v_i$$

where v_i has variance 1 and a correlation of v with u is ρ.

Derive and explain how to include a "Heckman correction" term in the linear regression for y_i in order to make the estimate of a unbiased. (You do not have to derive the inverse Mill's ratio. If you struggle with the math, you will get points if you can explain why we need a correction and what the correction does in order to fix the estimation problem.)

4. (15%) Prove that if G is the score of a likelihood function and H is the Hessian, then

$$E\{GG'\} = -E\{H\} \,.$$

5. (15%) Consider a sample of durations t_i observerved for $i = 1, ..., N$ of which some are incomplete spells. Assume the CDF for the duration has the form

$$1 - \exp\{-\theta t^{\alpha}\}$$

where θ and α are positive parameters.

a) Find the density, survivor, and hazard functions.

b) Write down the log-likelihood for the sample of observations.