**ECONOMETRICS II, Fall 2024**
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# Final Exam—November 20, 2024

Each sub-question in the following carries equal weight.

**1. (15%)** a) Explain how the simple bootstrap estimator works.
b) Explain what a parametric bootstrap is.

**2. (15%)** Assume that you have a sample of outcomes $y_i$, which can take values 0, 1, and 2, where the outcomes of $y$ indicate an ordering. You have a set of regressors $x_i$, that may help explain the outcomes.

a) What statistical model can you use to estimate such outcomes (give one example)?
b) Write down the log likelihood function for the model.

**3. (10%)** a) Write down an ARCH (or GARCH) model.
b) Write down a Stochastic Volatility model. (You do not have to write down exactly the one I studied in my Monte Carlo paper, but it has to have the basic idea.)

**4. (30%)**
Assume you are estimating the model

$$Y_i = aX_i + u_i \ ,$$

by OLS. Here $a$ is a scalar and we assume for simplicity that there is no intercept and that in the true underlying model (not censured or truncated) the error term has mean 0.
Assume that you only have 2 observations: $X' = (1, 2)$, $Y' = (2, 5)$.
The OLS estimate $\hat{a}$ is 2.4 and the residuals are 0.4 and –0.2 (they do not sum to zero because we have no constant).
a) (10%) Calculate the White robust standard error for $\hat{a}$.
b) (10%) Now assume that the two observations above form a group and we have a second group where (for computational simplicity) we also assume $X' = (1, 2)$, $Y' = (2, 5)$ . So your data are now $X' = (1, 2, 1, 2)$, $Y' = (2, 5, 2, 5)$. $\hat{a}$ is still 2.4 and the residuals as before (repeated).
b) (20%) Calculate the Robust standard error if you cluster on the two groups.

**5. 30%)** a) Assume you have estimated 3 parameters, with estimates $\hat{\beta}_1 = 7$, $\hat{\beta}_2 = 9$, and $\hat{\beta}_3 = 11$, and assume that you know for sure the estimates are normally distributed and the known variance-covariance matrix is

$$\Sigma = \begin{pmatrix} 5 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{pmatrix} .$$

a) Write down a (Wald) test statistic for ($\beta_2 = 2$, $\beta_3 = 1$). (I want you to write this as a scalar, a real number.)
b) Write down a test statistic for the hypothesis ($\beta_1 = 2$, $\log(\beta_1) - \beta_3^2 = 2$). (Again, I need to see the numbers.)
c) Assuming the coefficients are consistent, what is the distribution of the test statistics you wrote down?
d) What does the Matlab code below estimate?
e) In the Matlab code below, what is a test for the hypothesis ($beta1 = beta2$)? (You need to use the notation from the code, so I can see that you know.)
f) Explain in words (or how you would use the code, but not required here) how you could test ($beta1 = beta5$) using the output from the program.

## Final Exam Code 1.

This code estimates the simultaneous equation model

$$y_1 = \beta_0 + \beta_1 y_2 + \beta_2 x_1 + u_1 \ ,$$
$$y_2 = \beta_4 + \beta_5 y_1 + \beta_7 x_2 + u_2 \ ,$$

with $U \sim NID(0, \Sigma)$, using three stage least squares. Complete the code. You can write the formula, not using Matlab syntax, but you have to say what will do in terms of the output generated by the program. If you do not understand the code, you will not get points.)

## Set the parameters.

There are 1000 observations. Set $\beta_0 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.3$, $\beta_4 = 0.2$, $\beta_5 = 0.3$, $\beta_7 = 1$, $\sigma_1 = 1.2$, $\sigma_2 = 1.1$ and $\rho = 0.05$.

```
clear
clc

N = 1000;                              % Number of observations.

beta0 = 0.1;
beta1 = 0.5;
beta2 = 0.3;
beta4 = 0.2;
beta5 = 0.3;
beta7 = 1;
sigma1 = 1.2;
sigma2 = 1.1;
rho = 0.05;

mu = [0 0];                            % Mean vector of U.
smat = [sigma1^2 rho*sigma1*sigma2;    % Variance matrix of U.
        rho*sigma1*sigma2 sigma2^2];

G_inv = ones(2,2);                     % G^{-1} matrix.
G_inv(2,1) = beta1;
G_inv(1,2) = beta5;
G_inv = G_inv./(1-beta1*beta5);

B = zeros(3,2);                        % B matrix.
B(1,1) = beta0;
B(2,1) = beta2;
B(1,2) = beta4;
B(3,2) = beta7;
```

Generate the data.

Generate the data, $X$, then draw the error terms, $U$, and construct $Y$.

```
x1 = 2 + ((1:N)'/N).*normrnd(0,1,N,1);
x2 = 3 + 0.5*x1 + normrnd(0,1,N,1);
X  = [ones(N,1) x1 x2];              % X.

u = mvnrnd(mu,smat,N);               % U.

Y = X*B*G_inv + u;                   % Y.
Y1 = Y(:,1);
Y2 = Y(:,2);
```

## Estimation of a model.

Estimate the model using SOMETHING.

```
% Stage 1: .

b1_1 = (X'*X)\X'*Y1;
Y1_hat = X*b1_1;                    % Fitted Y1.

b2_1 = (X'*X)\X'*Y2;
Y2_hat = X*b2_1;                    % Fitted Y2.

% Stage 2.

X1_2 = [ones(N,1) Y2_hat x1];
X1X1 = X1_2'*X1_2;
X1Y1 = X1_2'*Y1;

b1_hat = (X1X1)\X1Y1;                % (eq. 1).

X2_2 = [ones(N,1) Y1_hat x2];
X2X2 = X2_2'*X2_2;
X2Y2 = X2_2'*Y2;

b2_hat = (X2X2)\X2Y2;                %  (eq. 2).
```

```
u1_2 = Y1-X1_2*b1_hat;          % Residuals, u1.
u2_2 = Y2-X2_2*b2_hat;          % Residuals, u2.
```