ECONOMETRICS II, Fall 2019

Bent E. Sørensen

Final Exam—December 2, 2019. Total number of points is 75%.

Each sub-question in the following carries equal weight except when otherwise noted.

1. (15%) Consider the AR(2) model

$$y_t = \mu + a\, y_{t-1} + b\, y_{t-2} + u_t \,,$$

where the error term is white noise with variance σ_u^2. Assume the model is stationary.

a) Find (derive) the mean of y.

b) Find the first order autocovariance. (It is a complete answer with the variance in the solution. You will find the variance in the next question.)

c) Find (derive) the variance.

2. (10%) Assume you have a model

$$Y_t = \mu + AX_t + u_t \,,$$

where $Y_t' = \{Y_t^1, Y_t^2\}$ are two variables (like income and consumption) and $\text{Var}\, Y_t = \Sigma$. Assume you have data for $t = 1, ..., T$, and the data are independent for different time periods. Assume you stack the data so that the full Y vector (and correspondingly for the columns of X), is $Y' = \{Y_1^1, ...Y_T^1, Y_1^2, ...Y_T^2\}$.

a) Show that the variance-covariance matrix for the residuals takes the forms $\Sigma \otimes I_T$. (Here, I_T is T-dimensional identity matrix.)

b) Show that if the data are stacked as $Y' = \{Y_1', ..., Y_T'\}$, then the variance-covariance matrix is $I_T \otimes \Sigma$.

3. (20%) Consider the model

$$y = \mu + u\,,$$

where u is $\mathrm{N}(0, \sigma^2)$.

a) Write down the likelihood function for a sample of N observations. Assume the observations are independent.
b) Find the gradient vector and the outer product of gradients. (You only need to do this for one observation. I am asking for $G_i G_i'$ in the notation of the ML-note.)
c) Find the Hessian (the matrix of second derivatives).
d) Show that the expectation of the outer product of gradients is equal to minus the expected value of the Hessian. (If you prefer, you can give a general proof, as in class, instead.)

4. (30%) At the end of the exam is the GMM code from one of your home works.

a) Explain how to change the code so that only lagged consumption is used as instruments.
b) The code for the weighting matrix is not included, but write down the Newey-West estimator for, say, a bandwidth of 3.
c) Explain how you would use the code to test if beta (the first parameter) takes a value of 0.98. You could use the LR-type test so a correct answer would explain the details of this, but you may also use a Wald test but again the full answer would explain the details (for full points you have to explain the parts from the code that would be used, for example, you can write down the formula for the test and then point out where the needed parts are in the code).

```
%{
    Xavier Martin G. Bautista
```

```
    Fall 2018
    Macro 3
    HW 2

    GMM_Main.m

    This replicates Hansen and Singleton (1982). Estimation is done using
    GMM. The variance matrix can be estimated using either Newey-West or
    Quadratic Spectral kernels.

    Note: Convention used is U(C) = (C^(1-gamma))/(1-gamma).
%}

%% 1. Change working directory and load data.

    close all
    clear
    clc

    addpath('D:/Xavier_Laptops/Xavier_Asus/Xavier_Classes/Fall_2018/Macro3/HW2')

    global c lag re rf n T Z

    load data

    lag = 3;                                                          % Number of

    c  = data(:,1);                                                   % c(t)/c(t-1
    re = data(:,2);                                                   % Value-weig
    rf = data(:,3);                                                   % T-bill rat
```

```
    T = size(data,1);
    Z = [ones(T-lag,1) c(1:T-3)  c(2:T-2) c(3:T-1)...                       % Instrument
                      re(1:T-3) re(2:T-2) re(3:T-1)...                      % value-weig
                      rf(1:T-3) rf(2:T-2) rf(3:T-1)];
    n = size(Z,2);                                                          % Number of

    clear data

%% 2. GMM Stage 1: Identity Weighting Matrix.

    b0 = [0.5 0.5];                                                         % Initial gu
    W  = weight(b0,0);                                                      % 0 = Identi

    opt = optimset('FinDiffType','central','HessUpdate','BFGS');            % Use centra
    b1  = fminunc('gmm_obj',b0,opt,W);                                      % Weighting

    clear W

%% 3. GMM Stage 2: Optimal Weighting Matrix.

    % Newey-West.

    W   = weight(b1,1);
    bNW = fminunc('gmm_obj',b1,opt,W);                                      % GMM Estima

    gradNW = Df(bNW);                                                       % Gradient.
    vmatNW = inv(gradNW'*W*gradNW);                                         % Variance m

    clear gradNW W Dgp1 Dgp2 Dgm1 Dgm2
```

```
    % Quadratic Spectral.

    W   = weight(b1,2);
    bQS = fminunc('gmm_obj',b1,opt,W);                                          % GMM Estima

    gradQS = Df(bQS);                                                           % Gradient.
    vmatQS = inv(gradQS'*W*gradQS);                                             % Variance m

    clear gradQS W Dgp1 Dgp2 Dgm1 Dgm2

%% 4. Print results.
    clc
 XXX print commands left out

 function ZXb = orth(guess)

%{
    orth.m

    This is the orthogonality condition from Hansen and Singleton (1982)
    E(z(t)*((beta*((C(t)/C(t-1))^(-gamma))*r(t)) - 1)) = 0.
%}

global c lag n rf T Z

beta  = guess(1);
gamma = guess(2);

C = repmat(c(1+lag:T),1,n);
```

```
R = repmat(rf(1+lag:T),1,n);

ZXb = Z.*((beta.*(C.^(-gamma)).*R)-1);

end

function crit = gmm_obj(guess,W)

%{
    gmm_obj.m

    This is the quadratic GMM objective function for Hansen and Singleton
    (1982).
%}

global lag T

mom = ((sum(orth(guess),1))./(T-lag))';
crit = mom'*W*mom;

end
```