ECONOMICS 7344, part 2, Spring 2022
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Final Exam, May 4–4 questions, total weight is 100%, all sub-questions carry equal weight, except when otherwise noted.

1. (20%) Assume that income follows the MA process

$$y_t = 3 + \Sigma_{k=0}^{5} e_{t-k} \, ,$$

with realizations $e_{t-i} = 10$ for $i = 0, ..., 10$.

i) What is $E_t y_{t+1}$?
ii) What is $E_t \Delta y_{t+1}$?

For the next question assume consumption is determined by the PIH with interest and discount rates equal to 10%.

iii) What is the change in consumption ΔC_t?

2. (20%) A consumer lives forever and has assets 0 and income 10 in period 0, discount and interest rates of 10%, and quadratic utility. With probability 1/2 the consumer has income 10 in period 1 and with probability 1/2 the consumer has income 20 in period 1. The income in all periods after period 1 is the same as in period 1. (Note: this is supposed to be the harder question, so remember that you get points for addressing the problem coherently whether you get to the end or not.)

i) Find C_0
ii) Find the distribution of C_1. (This means seen from period 0.)
iii) Find the distribution of C_2.

3. (20%). Assume that there are two states of the economy next year, "good" and "bad," each with probability 0.5. In the good state aggregate consumption grows 4% and in the bad state it grows 0%. Now consider assets D and E. For these we know the payouts. For D the payout is 5 in the bad state and 15 in the good state, while for E the payout is 5 in the bad state and 5 in the good state. Use the CCAPM as it was derived in the handout. The safe rate of return is 1%.
a) What would be the prices of assets D and E?
b) What would be the returns (you can give gross or net, but state which) of assets D and E?

4. (40%). Assume an economy consists of N agents who maximize a von Neumann-Morgenstern utility function

$$U(C_0) + E_0 U(C_1) ,$$

where $U(C_t) = -\frac{1}{\gamma}e^{-\gamma C_t}$ where γ is a positive constant. Assume there is no storage and perfect Arrow-Debreu markets. There are two time periods ($t = 0$ and $t = 1$) and two states of the world "A" and "B" in period 1. Assume there are N consumers in the economy.

a) Demonstrate (do the derivations) that this economy allows for a representative agent.
b) Derive a formula for the relation between the consumption of each agent and aggregate consumption.

Now assume that $N = 2$ and the endowment of the first agent is $y_0^1 = 3$, $y_1^1 = 5$ in state A and $y_1^1 = 1$ in state B. The endowment of the second agent in period 0 is $y_0^2 = 3$ and in period 1 his or her endowment is $y_1^2 = 1$ in state A and $y_1^2 = 5$ in state B. Assume that state A happens with probability 1/2. Set $\gamma = 2$.

c) For now assume that agents can trade in a bond but no other financial assets exist. Find the rate of interest.
d) Explain why the rate of interest is positive or negative using concepts from the class.
e) Now assume that the agents can trade in Arrow securities for state A and state B. Find the prices of the Arrow securities and the rate of interest. Explain why is it higher or lower than the rate of interest you found in question c).
f) Under the assumptions of part b, find the consumption of the "home" agent in each period and in each state of the world.