

**Final Exam, April 30–5 questions, total weight is 100%, all sub-questions carry equal weight, except when otherwise noted.**

1. Assume that income follows the ARMA process

$$y_t = 3 - .6y_{t-1} + e_t - e_{t-1},$$

and consumption is determined by the PIH.

- i) (10%) What would be the change in consumption following a 100\$ innovation to income assuming an interest rate of 10%?
- ii) (20%) Derive the formula you used in part i). (There is a number of steps in the proof, you get partial points for each.)

2. (10%) A consumer lives for 3 periods and earns 1\$, 2\$, and 4\$ in periods 1, 2, and 3, respectively. The consumer maximizes

$$U(C_1) + E_1U(C_2) + E_1U(C_3),$$

where

$$U(C) = -\frac{1}{2}C^{-2}.$$

The consumer can freely borrow and lend at a known interest rate. The rate of interest is 10 percent.

- a) Find the growth rate of consumption from period 1 to period 2.
- b) Find the level of consumption in each period. (If you correctly state the equations [using numbers] that determine the solution, you do not need to solve them.)

3. (10%) Explain what is meant by the Equity Premium Puzzle. You should state clearly under which assumptions there is a puzzle. (You are not asked to derive the expression that I derived in class, but you would need to know what it is.)

4. (15%) A consumer lives for 2 periods and earns  $Y_1 = 10\$$ , in period 1, and in period 2 he or she earns  $Y_2^a = 10\$$  with probability 1/2 (state  $a$ ) and  $Y_2^b = 30\$$  with probability 1/2 (state  $b$ ). The consumer starts with 0 assets and maximizes

$$U(C_1) + E_1U(C_2),$$

where

$$U(C) = \log(C).$$

Assume that the safe rate of interest is 10 percent.

Assume that the consumer has access to a safe bond, of which he or she buys an amount  $B$ , that has a return of 10% and that a stock (equity) exists besides the safe bond. Let the amount of equity bought be  $S$  (it can be negative). Assume that the stock has a (net) rate of return of 0% if state  $a$  occurs [meaning that agent gets back the principal] and 100% if state  $b$  occurs. Find  $B$  and  $S$  and the implied consumption plan; that is, consumption in each period and state-of-the-world. (Note: the question is set up with “extreme” values to make the algebra easier, so the solution may also be “extreme.” If you are running out of time, most points will be accrued when you write down the equations (with numbers) that determines the answer.)

**5. (35%)** Assume an economy consists of two agents (home and foreign) who maximize a von Neumann-Morgenstern utility function

$$U(C_0) + E_0U(C_1) ,$$

where  $U(C_t) = -\frac{1}{\gamma} \exp(-\gamma C_t)$  where  $\gamma$  is a positive constant. Assume there is no storage and perfect Arrow-Debreu markets. There are two time periods ( $t = 0$  and  $t = 1$ ) and two states of the world “A” and “B” in period 1. Assume there are  $N$  consumers in the economy.

- a) Demonstrate (do the derivations) that this economy allows for a representative agent.
- b) Derive a formula for the relation between the consumption of each agent and the aggregate consumption

Now assume that  $N = 2$ ,  $\gamma = 1$ , and the endowment of the first agent is  $y_0^1 = 3$ ,  $y_1^1 = 5$  in state A and  $y_1^1 = 1$  in state B. The endowment of the second agent in period 0 is  $y_0^2 = 3$  and in period 1 his or her endowment is  $y_1^2 = 1$  in state A and  $y_1^2 = 5$  in state B. Assume that state A happens with probability 1/2.

- c) For now assume that agents can trade in a bond but no other financial assets exist. Find the rate of interest.
- d) Explain why the rate of interest is positive or negative using concepts from the class.
- e) Now assume that the agents can trade in Arrow securities for state A and state B. Find the prices of the Arrow securities and the rate of interest. Explain why is it higher or lower than the rate of interest you found in question c).
- f) Find the consumption of each agent in each period and in each state of the world. (Hint: Use the result from part b).)